First Semester B.E.

Engineering Mathematics – I

Model Question Paper – I

Note: Answer any five full questions choosing at least two full questions from each part.

Part – A

1. (a) Find the n[th] derivatives of

   (i) $e^{-x}\sin^2 x$ (ii) $\frac{x}{(x-1)(2x+3)}$ **6**

   (b) If $Cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, then prove that

   $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$ **7**

   (c) Find the pedal equation of the curve

   $r^n = a^n \cos n\theta + b^n \sin n\theta$ **7**

2. (a) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$ **6**

   (b) If $u = f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **7**

   (c) If $x = u\cos v, y = u\sin v$, show that $JJ^1 = 1$ **7**

3. (a) Obtain the reduction formula for $I_n = \int_0^{\pi/2} \sin^n x dx$ where n is a positive integer and hence evaluate I_4 **6**

   (b) Evaluate: $\int_0^{2a} x^3\sqrt{2ax - x^2}\, dx$ **7**

   (c) Trace the curve: $x^{2/3} + y^{2/3} = a^{2/3}$ **7**

4. (a) For the Cycloid $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$, find $\frac{ds}{dx}, \frac{ds}{dy}$ **6**

   (b) Find the volume of the solid generated by revolving the Lemniscate $r^2 = a^2\cos 2\theta$ about the line $\theta = \frac{\pi}{2}$ **7**

   (c) Evaluate: $\int_0^1 \frac{x^\alpha - 1}{\log x} dx;$ given $\alpha \geq 0$ **7**

Part – B

5. (a) Solve any two

(i) $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$ **7**

(ii) $xy(1+xy^2)\frac{dy}{dx} = 1$ **7**

(iii) $\left[y(1+\frac{1}{x}) + \cos y\right]dx + \left[x + \log x - x\sin y\right]dy = 0$ **7**

(b) Find the orthogonal trajectories of the family of Confocal Conics $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, Where λ is the parameter **6**

6. (a) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}$ **6**

(b) Test for convergence:

$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \cdots + \frac{2^n - 2}{2^n + 1}x^{n-1} + \cdots; x > 0$ **7**

(c) Define Absolute Convergence and Conditional Convergence. Is the following series Absolutely Convergent?

$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{2} + \cdots$ **7**

7. (a) Show that the lines whose direction cosines satisfy the equation $l + m + n = 0, \quad 2l^2 + 2m^2 - n^2 = 0$ are parallel. **6**

(b) Find the equation of the plane passing through the line of intersection of the planes $7x - 4y + 7z + 16 = 0$ & $4x + 3y - 2z + 13 = 0$ and perpendicular to the plane $x - y - 2z + 5 = 0$ **7**

(c) Find the distance of the point $(3,-4,5)$ from the plane $2x + 5y - 6z = 16$ measured parallel to the line $\frac{x}{2} = \frac{y}{1} = \frac{z}{-2}$ **7**

OR

Find the magnitude & equation of the shortest distance between $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-2}{-5} = \frac{z+2}{2}$ **7**

8. (a) A particle moves on the curve $x = 2t^2,\ y = t^2 - 4t,\ z = 3t - 5,$ where t is the time. Find the components of velocity & acceleration at time $t = 1$ in the direction $\vec{i} - 3\vec{j} + 2\vec{k}$ **6**

(b) Find the directional derivative of $\phi = x^2 yz + 4xz^2$ at the point $(1,2,-1)$ in the direction of the vector $2\vec{i} - \vec{j} - 2\vec{k}$ **7**

(c) Prove that $Curl\,(grad\,\phi) = \vec{0}$ **7**

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First semester B E

Engineering mathematics -1

Model question paper -2

Note: Answer any five full questions choosing at least two full questions.

PART-A

1. a) Find the nth derivative of $x^4 + \log_{10}\left(3x^2 + 5x - 2\right)$ **6**

   b) If $y = e^{\tan^{-1} x}$, prove that $\left(1+x^2\right)y_{n+2} + \left[2(n+1)x - 1\right]y_{n+1} + n(n+1)y_n = 0$ **7**

   c) With the usual notation, prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$ **7**

2. a) If $z(x+y) = x^2 + y^2$, show that $\left[\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]^2 = 4\left[1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right]$ **6**

   b) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + z\frac{\partial z}{\partial z} = 0$ **7**

   c) Find the error in the area of an ellipse if 1% error is

   measuring the major and minor axes. **7**

3. a) Obtain a reduction formula for $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$ and hence evaluate I_5

   6

   b) Evaluate: $\int_0^1 x^4\left(1-x^2\right)^{\frac{3}{2}} dx$ **7**

   c) Trace the curve: $r^2 = a^2 \cos 2\theta$ **7**

4. a) With usual meanings for r, s, θ *and* ϕ for the polar curve $r = f(\theta)$, show that

   $\frac{d\phi}{d\theta} + r \operatorname{cosec}^2\theta \frac{d^2 r}{ds^2} = 0$ **6**

   b) Find the entire length of the cardioid $r = a(1 + \cos\theta)$ **7**

   c) Evaluate $\int_0^{\pi} \log(1 + a\cos x)dx$, using the method of differentiation. **7**

   under the Integral sign

PART-B

5. a) Solve any two:

i) $x^4 \frac{dy}{dx} + x^3 y + \operatorname{cosec} xy = 0$ **7**

ii) $(1 + y^2)dx = (\tan^{-1} y - x)dy$ **7**

iii) $(5x^4 + 3x^2 y^2 - 2xy^3)dx + (2x^3 y - 3x^2 y^2 - 5y^4)dy = 0$ **7**

b) Find the orthogonal trajectories of the family of the curves

$r^n = a^n \sin n\theta$ **6**

6. a) Test the convergence of the series: $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$ **6**

b) Test for convergence of the series $\sum \frac{4.7......(3n+1)}{1.2........n} x^n$ **7**

c) Discuss the convergence of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + -..........$ **7**

7. a) Find the angle between any two diagonals of a cube. **6**

b) Find the equation of the plane passing through the points $(2,2,1)$ and $(9,3,6)$ and perpendicular to the plane $2x + 6y + 6z = 9$ **7**

c)Find the point of intersection of the lines $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ and **7**

$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ Also, find the equation of the plane containing them.

OR

Find the magnitude and equations of the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \quad and \quad \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$ **7**

8. a) A particle move along a curve $x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t.$ find the velocity and acceleration and their magnitudes at $t = 0$ **6**

b) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ **7**

c) Prove that $div(\vec{A} * \vec{B}) = \vec{B}.curl\vec{A} - \vec{A}.curl\vec{B}$ where $\vec{A}$ *and* $\vec{B}$ are vectors differentiable functions. **7**

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