USN | | | | | | | | | | 10MAT11

First Semester B.E. Degree Examination, June/July 2011
Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART - A

1 a. i) If $y = (ax+b)^{-1}$, then y^n is (04 Marks)

A) $\dfrac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ B) $\dfrac{(-1)^n n!a^n}{(ax+b)^{n+1}}$ C) $\dfrac{n!a^n}{(ax+b)^{n+1}}$ D) Zero

ii) The Taylor's theorem relates the value of the function and its
A) I^{st} order derivative B) II^{nd} order derivatives
C) Constant D) Higher order derivatives

iii) Cauchy's mean value theorem reduces to Lagrange's mean value theorem, if
A) $f(x) = g(x)$ B) $f'(c) = g'(c)$ C) $g(x) = x$ D) $f(x) = 0$

iv) To find the n^{th} derivative of a function $y = f(x)$, its (n-1) derivatives must be a
A) function of y B) function of x
C) constant D) function of x & y

b. If $\cos^{-1}\left(\dfrac{y}{b}\right) = \log\left(\dfrac{x}{n}\right)^n$ Prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$ (06 Marks)

c. Verify Lagrange's mean value theorem for the function $f(x) = \log x$ in the interval [1, 2] and find the value of 'C'. (04 Marks)

d. Expand tanx in powers of $(x - \pi/4)$ upto third degree term. (06 Marks)

2 a. i) L Hospital's rule implies that each differentiation reduces the order of the infinitesimals by
A) unity B) two C) zero D) four

ii) If two curve cuts orthogonally, then angle between their tangents is equal to
A) zero B) $\pi/4$ C) $3\pi/4$ D) $\pi/2$

iii) Perpendicular distance from the pole on the tangent is equal to
A) Sin ϕ B) cos ϕ C) r sin ϕ D) r cos ϕ

iv) The value of radius of curve remains unchanged under the change of
A) ordinates B) signs C) derivatives D) none of these (04 Marks)

b. Evaluate $\underset{x\to 0}{Lt}\left(\dfrac{1}{x} - \cot x\right)$. (04 Marks)

c. Prove that the radius of curvature ρ at any point (x, y) on the curve $\sqrt{\dfrac{x}{a}} + \sqrt{\dfrac{y}{b}} = 1$ is given by $\rho = \dfrac{2(ax+by)^{3/2}}{ab}$. (06 Marks)

d. Find the pedal equation of the curve $\dfrac{2a}{r} = (1+\cos\theta)$. (06 Marks)

3 a. i) A partial increment corresponds to a change of one of the variables and all other variables are **(04 Marks)**
A) constant B) varying C) incremented D) decremented
ii) If $x = r\cos\theta$, $y = r\sin\theta$, the Jacobian of (x, y) with respect to (r, θ) is equal to
A) $\frac{1}{r}$ B) θ C) r D) zero
iii) The necessary conditions for $f(x, y)$ to have a maximum or minimum at (a, b).
A) $f_x(a, b) = 0$ B) $f_y(a, b) = 0$
C) $f_{xy}(a, b) = 0$ D) $f_x(a, b) = f_y(a, b) = 0$
iv) In Lagrange's method of undetermined multipliers are cannot determine the nature of the
A) Function B) Stationary point C) Multipliers D) None of these
b. Find the extreme value of the function $f(x) = x^3 + y^3 - 3axy$, $a > 0$. **(06 Marks)**
c. If $u = \log(x^3 + y^3 + z^3 - 3xy)$ show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ **(06 Marks)**
d. Find the percentage error in the area of an ellipse when an error of 1% is made in measuring the major and minor axis. **(04 Marks)**

4 a. i) Any motion in which the curl of the velocity vector is zero is said to be **(04 Marks)**
A) rotational B) solenoidal C) irrotational D) conservative
ii) The directional derivative of a scalar function ϕ at any point is ______ along $\nabla\phi$.
A) minimum B) maximum C) zero D) ∞
iii) Gradient of a scalar field is a
A) constant B) scalar C) vector D) None of these
iv) If $\phi(x, y, z) = c$ is the equation of surface, then $\nabla\phi$ is ______ to the surface.
A) parallel B) normal C) inclined D) not parallel
b. Find the constants a, b, c so that the vector function
$\hat{F} = (x + 2y + az)\,\hat{i} + (bx - 3y - z)\,\hat{j} + (4x + (y + 2z)\,\hat{k}$ is irrotational **(04 Marks)**
c. Prove that grad div F = curl curl F + ∇^2 F. **(06 Marks)**
d. Show that the spherical co-ordinate system is orthogonal. **(06 Marks)**

PART - B

5 a. i) Any integral formula which express in terms of another similar integral in lower powers is called ______ formula **(04 Marks)**
A) integral B) differential C) reduction D) trigonometric
ii) If given equation contains only even powers of x, then the curve is symmetrical about
A) y – axis B) x –axis C) both axis D) None of these
iii) Surface of solid generated by revolution about x –axis of the curve $y = f(x)$ between $x = a$, $x = b$.
A) $\int_a^b \pi y^2\,dx$ B) $\int_a^b \pi x\,dy$ C) $\int_a^b \pi r^2\,d\theta$ D) $\int_a^b 2\pi y\,ds$
iv) Leibniz's rule for differentiation under integral sign is
A) $\phi'(y) = \int_a^b \frac{\partial}{\partial y} f(x,y)dx$ B) $\phi'(y) = \int_a^b \frac{\partial}{\partial x\partial y} f(x,y)dx$
C) $\phi(y) = \int_a^b \frac{\partial}{\partial x} f(x,y)dx$ D) None of these
b. Obtain the reduction formula for $\int \cos^n x\,dx$. **(06 Marks)**

c. Evaluate $\int_0^{\pi/2} \sin^7\theta \cos^6\theta \, d\theta$. **(04 Marks)**

d. Find the volume of the solid obtained by revolving the Astraid $x^{2/3} + y^{2/3} = a^{2/3}$ about x – axis. **(06 Marks)**

6 a. i) The degree of the differential equation $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}} = C$ is **(04 Marks)**

A) Two B) Three C) One D) Zero

ii) Variable separable form of the equation $\dfrac{y}{x}\dfrac{dy}{dx} = \sqrt{1+x^2+y^2+x^2y^2}$ is

A) $\dfrac{\sqrt{1+y^2}}{y}dy = \dfrac{\sqrt{1+x^2}}{x}dx$ B) $\dfrac{y}{\sqrt{1+y^2}}dy = x\sqrt{1+x^2}\,dx$

C) $\sqrt{1+x^2}\,dx + \sqrt{1+y^2}\,dy$ D) $\dfrac{y}{\sqrt{1+y^2}}dy = \dfrac{x}{\sqrt{1+x^2}}dx$

iii) The integrating factor of the differential equation $x \log x \dfrac{dy}{dx} + y = \log x^2$ is

A) $\log x^2$ B) $\log x$ C) $x \log x$ D) $x \log x^2$.

iv) The differential equation $(x + x^8 + ay^2)\,dx + (y^8 - y + bxy)\,dy = 0$ is exact if b = ___

A) 4 B) 3x C) 2a D) 4a

b. Solve $\dfrac{dy}{dx} = \dfrac{y}{x - \sqrt{xy}}$. **(04 Marks)**

c. Solve $\dfrac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$. **(06 Marks)**

d. Find the orthogonal trajectories of the family of curve $r^n \cos n\theta = a^n$. **(06 Marks)**

7 a. i) The normal form of the matrix of rank r is **(04 Marks)**

A) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ii) If rank of the coefficient matrix is equal to rank of the Augmented matrix then equations are

A) consistence B) inconsistence

C) have no solution D) have infinite number of solutions.

iii) In Gauss – elimination method coefficient matrix reduces to ____ matrix.

A) diagonal B) unit matrix C) triangular D) None of these

iv) The system of linear homogeneous equations have trivial solution if all variable are (i = 1 … n)

A) $x_i > 0$ B) $x_i < 0$ C) $x_i = 0$ D) $x_i = \infty$

b. Investigate the value of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have i) unique solution ii) no solution iii) infinite number of solutions. **(06 Marks)**

c. Solve using the Gauss – Jordan method,

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

(06 Marks)

d. Find the rank of the Matrix of $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 1 & 2 & 3 & 4 \end{bmatrix}$ (04 Marks)

8 a. i) Each eigen vector corresponding to a eigen value is (04 Marks)

A) unique B) no unique C) infinite D) None of these

ii) The sum of the eigen values of the matrix is the sum of the elements of

A) Any row B) Any column

C) diagonal D) Any row and column.

iii) A homogeneous expression of the second degree in any number of variables is called

A) linear form B) cubic form C) quadratic form D) None of these

iv) Every square matrix satisfies its own _______ equation.

A) quadratic B) cubic C) algebraic D) characteristic

b. Find the characteristics equation and eigen vector of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

(06 Marks)

c. Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form using characteristic equation method. (06 Marks)

d. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2zy$ to the canonical form. (04 Marks)

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