USN | | | | | | | | | | MAT11

First Semester B.E. Degree Examination, December 2010
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions, selecting at least ONE question from each part.*

PART – A

1 a. A line makes angles α, β, γ and δ with the four diagonals of a cube, prove that : $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$. **(07 Marks)**

b. Find the equation of the plane through the point (-1, 2, 4) and parallel to the plane $2x - 3y - 5z + 6 = 0$. **(07 Marks)**

c. Find the equation to the line through the point (1, 2, 3) and parallel to the line whose equations are $x - y + 3z = 5$, $3x + y + z = 6$. **(06 Marks)**

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2 a. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. **(07 Marks)**

b. Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. **(07 Marks)**

c. Find the equation of the right circular cone whose vertex is the origin, semi vertical angle is 45° and whose axis is the line $x = 2y = z$. **(06 Marks)**

PART – B

3 a. Find the n^{th} derivative of $\frac{x^2}{(x+2)(2x+3)}$. **(07 Marks)**

b. Find the angle between radius vector and tangent for the curve $r = a(1 - \cos\theta)$. **(07 Marks)**

c. Prove that with usual notations $\frac{1}{p^2} = u^2 + \left(\frac{du}{d\theta}\right)^2$. **(06 Marks)**

4 a. If $z = f(x, y)$ is a homogeneous function of x and y of degree n, then prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$. **(07 Marks)**

b. If $H = f(y - z, z - x, x - y)$, prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$. **(07 Marks)**

c. Find the percentage error in the area of an ellipse when an error of +1% is made by measuring the major and minor axis. **(06 Marks)**

PART – C

5 a. Obtain reduction formula for $\int \text{Sin}^n x dx$, $x \geq 0$. **(07 Marks)**

b. Trace the curve $y^2(a - x) = x^3$. **(07 Marks)**

c. Evaluate $\int_0^{\pi} x \,\text{Sin}^5 x \, dx$. **(06 Marks)**

6 a. Trace the curve $r = a \,\text{Cos}\, 2\theta$. **(07 Marks)**

b. Show that the area enclosed between parabolas $y^2 = 4a\,(x + a)$ and $y^2 = -\,4a\,(x - a)$ is $\frac{16}{3}a^2$. **(07 Marks)**

c. Find the volume of the solid formed by the revoluation of one arch of the cycloid $x = a\,(\theta - \text{Sin}\,\theta)$, $y = a\,(1 - \text{Cos}\,\theta)$ about its base. **(06 Marks)**

PART – D

7 a. Solve $\frac{dy}{dx} = \frac{x + 2y + 3}{2x - y + 1}$. **(07 Marks)**

b. Solve $\frac{dy}{dx} + y \sec x = \tan x$. **(07 Marks)**

c. Solve $(1 + e^{x/y})\,dx + (1 - x/y)\,e^{x/y}\,dy = 0$. **(06 Marks)**

8 a. Test the convergence of $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \ldots\ldots (x > 0)$. **(07 Marks)**

b. Test the convergence of the series $\frac{x}{3} + \frac{1.2}{3.5}x^2 + \frac{1.2.3}{3.5.7}x^3 + \ldots.$ **(07 Marks)**

c. Discuss the convergence of the series $1 - 2x + 3x^2 - 4x^3 + \ldots.\ (0 < x < 1)$. **(06 Marks)**

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