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## Seventh Semester B.E. Degree Examination, June/July 2011 Operations Research

Time: 3 hrs.
Max. Marks:100

## Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

## PART-A

1 a. "Operations research is more than just mathematics". Justify the statement, with an example.
(03 Marks)
b. List and explain the steps in conducting an 'operations research' study.
(05 Marks)
c. A plant manufactures two products A and B . The profit contribution of each product has been estimated to be Rs. 20 and Rs. 24 for products A and B respectively. Each product passes through two departments of the plant. The time required for each product and the total time available in each department are as follows :

| Department | Time (hrs) required/unit of |  | Available time <br>  <br>  <br> (hrs) per month |
| :---: | :---: | :---: | :---: |
|  | 2 | 3 | 1500 |
| 2 | 3 | 2 | 1500 |

The plant has to supply the products to market where the maximum demand for product B is 450 units/month. Formulate the problem as an LP model and find graphically, the number of products A and B to maximize the total profit per month.
(12 Marks)

2 a. Obtain the dual of the following primal LP problem.
Minimize $Z=3 x_{1}-2 x_{2}+x_{3}$
Subject to $\quad 2 x_{1}-3 x_{2}+x_{3} \leq 5$

$$
4 x_{1}-2 x_{2} \geq 9
$$

$$
-8 x_{1}+4 x_{2}+3 x_{3}=8
$$

$x_{1}, x_{2} \geq 0, x_{3}$ unrestricted in sign.
(05 Marks)
b. Solve the following LP problem by $\mathrm{Big}-\mathrm{M}$ method.

Maximize $Z=2 x_{1}+3 x_{2}+4 x_{3}$
Subject to $\quad 3 x_{1}+x_{2}+4 x_{3} \leq 600$
$2 x_{1}+4 x_{2}+2 x_{3} \geq 480$
$2 \mathrm{x}_{1}+3 \mathrm{x}_{2}+3 \mathrm{x}_{3}=540$
$\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3} \geq 0$.
(15 Marks)
3 a. Distinguish between : i) a balanced and an unbalanced transportation problem ; ii) a transportation problem and an assignment problem.
(02 Marks)
b. Obtain the initial solution by VAM and optimal solution by MODI method for the transportation problem shown below :

Unit transportation cost (Rs.) to markets

| \% |  | A | B | C | Supply |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| ت | $\mathrm{W}_{1}$ | 5 | 4 | 6 | 65 |
| $\frac{\bar{\pi}}{0}$ | $\mathrm{W}_{2}$ | 7 | 4 | 7 | 42 |
| 辰 | $\mathrm{W}_{3}$ | 8 | 6 | 7 | 43 |
| Dem | and - | 70 | 30 | 50 |  |

(08 Marks)
c. A solicitors firm would like to employ typists on hourly piece-rate basis for their work. There are five typists and their charges and capability (typing speed) are different. There are five jobs available with the firm and one job is to be assigned to one typist. A typist is paid for full hours even if he/she works for a fraction of an hour. Find an optimum assignment of typists to jobs to minimize the total cost to the firm, given the following data :

| Typist | Rate/hr, Rs. | No. of pages typed/hr. |
| :---: | :---: | :---: |
| A | 5 | 12 |
| B | 6 | 14 |
| C | 3 | 8 |
| D | 4 | 10 |
| E | 4 | 11 |


| Job | Number of pages |
| :---: | :---: |
| P | 199 |
| Q | 175 |
| R | 145 |
| S | 298 |
| T | 178 |

(10 Marks)
4 a. List the assumptions made while dealing with sequencing problems.
(03 Marks)
b. There are seven jobs, each of which has to be processed through two machines in the order M-1 M-2. Processing time in minutes are given as

| Job | A | B | C | D | E | F | G |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Machine $\mathrm{M}_{1}$ | 3 | 12 | 15 | 6 | 10 | 11 | 9 |
| Machine $\mathrm{M}_{2}$ | 8 | 10 | 10 | 6 | 12 | 1 | 3 |

Determine a sequence of the jobs that will minimize the total elapsed time. Also, find the total elapsed time, and idle time for machine $\mathrm{M}_{1}$ and machine $\mathrm{M}_{2}$. Show the sequence on a Gantt chart.
(08 Marks)
c. Use graphical method to minimize the time required to process the two jobs on five machines. For each machine, specify the job which should be done first. Find the total elapsed time to complete both jobs.

| Job 1 | Sequence : | A | B | C | D | E |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Time (hr) : | 6 | 8 | 4 | 12 | 4 |
| Job 2 | Sequence : | B | C | A | D | E |
|  | Time (hr) : | 10 | 8 | 6 | 4 | 12 |

(09 Marks)

## PART - B

5 a. Explain the need for studying queues.
(05 Marks)
b. At the balcony ticket counter of a cinema hall, customers arrive at the rate of 12 per hour according to Poisson distribution. The single clerk at the counter serves the customers at the rate of 30 per hour.
i) What is the probability that there is no customer in the counter?
ii) What is the probability that there are more than 2 customers in the counter?

Find:
i) Average number of customers in the system (counter) and in the queue.
ii) Average time a customer spends in the system and in the queue.
(07 Marks)
c. Goods trucks arrive randomly at a stockyard with a mean of 8 trucks/hour. A crew of four operatives can unload a truck in 6 min . Trucks waiting in queue to be unloaded are paid a waiting charge at the rate of Rs.60/hour. Operatives are paid a wage rate of Rs.20/hour. It is possible to augment the crew strength to 2 or 3 (of four operatives per crew) when the unloading time will be 4 minutes or 3 minutes respectively per truck. Find the optimal crew size.
(08 Marks)
6 a. With sketches, give the meaning of merge and burst events as applicable to a project network.
(02 Marks)
b. A small project consists of six activities. The duration (in days) of each activity and their immediate predecessors are shown below :

| Activity | A | B | C | D | E | F |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Immediate predecessor | - | - | - | A, B | B | B,C |
| Duration (days) | 5 | 3 | 7 | 8 | 4 | 5 |

i) Draw the network
ii) Find the critical path
iii) Verify the critical path by earliest time and latest time values.
(09 Marks)
c. The activities in a project are defined by three time estimates - optimistic, most likely, and pessimistic - the values of which are shown below :

| Activity | $1-2$ | $2-3$ | $2-4$ | $3-5$ | $4-5$ | $5-6$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{t}_{\mathrm{o}}$, days | 1 | 1 | 1 | 1 | 2 | 2 |
| $\mathrm{t}_{\mathrm{m}}$, days | 2 | 4 | 2 | 2 | 3 | 3 |
| $\mathrm{t}_{\mathrm{m}}$, days | 3 | 7 | 9 | 9 | 4 | 4 |

Draw the network and determine, i) expected completion time of each activity; ii) expected completion time of the project ; iii) probability of completing the project in 14 days (for a ' $z$ ' value of 1.6 , area under normal curve is 0.877 ).
(09 Marks)

7 a. Solve the following game by sing the principle of dominance :

Player A

| Player B |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | I | II |  | IV | V | VI |
| 1 | 4 | 2 | 0 | 2 | 1 | 1 |
| 2 | 4 | 3 | 1 | 3 | 2 | 2 |
| 3 | 4 | 3 | 7 | -5 | 1 | 2 |
| 4 | 4 | 3 | 4 | -1 | 2 | 2 |
| 5 | 4 | 3 | 3 | -2 | 2 | 2 |

(10 Marks)
b. Reduce the following game to ' $3 \times 2$ ' by dominance principle and then solve it by graphical method.

Player B

(10 Marks)

8 a. Solve the following linear programming problem by Gonory technique :
Maximize $Z=3 x_{2}$
Subject to $3 \mathrm{x}_{1}+2 \mathrm{x}_{2} \leq 7$
$-x+x_{2} \leq 2$
$x_{1}, x_{2} \geq 0$ and are integers.
(12 Marks)
b. Úse branch and bound method to solve the following linear programming problem :

Minimize $Z=4 x_{1}+3 x_{2}$
Subject to $5 x_{1}+3 x_{2} \geq 30$
$\mathrm{x}_{1} \leq 4$
$x_{2} \leq 6$
$x_{1}, x_{2} \geq 0$ and are integers.

