Spur gear

1. Derive Lewis equation for strength of gear teeth and state the assumptions made
2. Differentiate between spur and helical gears
3. A pair of carefully cut gears with 20^0 FDI profile is used to transmit 12 kW at 1200 rpm of pinion. The gear has to rotate at 300 rpm. The material used for both pinion and gear is medium carbon steel whose allowable bending stresses may be taken as 230 MPa. Determine the module and face width of spur pinion and gear. Check the pair for wear strength against dynamic load. Take 24 teeth on pinion. Assume modulus of elasticity as 202 GPa. **(Dec.09/Jan. 10)(Marks 15)**

4. A spur pinion of cast steel $(\sigma_d = 140 \quad MN/m^2)$ is to drive a spur gear of cast Iron $(\sigma_d = 55 \quad MN/m^2)$. The transmission ratio is to be $2\frac{1}{2}$ ti 1. The diameter of the pinion is to be 105 mm & the teeth are full depth involutes form. Design for the greatest number of teeth. Determine the necessary module and face width of the gears for strength only. Power to be transmitted is 20 kW at 900 rpm of the pinion. **(Dec.08/Jan. 09)(Marks 15)**

5. A pair of spur gears has to transmit 20 kW from a shaft rotating at 1000 rpm to a parallel shaft which is to rotate at 310 rpm. Number of teeth on pinion is 31 with 20^0 full depth involute tooth form. The material for pinion is steel C40 untreated with allowable static stress 207 MPa and the material for gear is cast steel 0.20% C untreated with allowable static stress 138.3 MPa. Determine the module and face width of the gear pair. Also find dynamic tooth load on the gears. Take the service factor as 1.5 and check the wear resistance of the gear. **(May/June 2010)(Marks 20)**

6. A pair of straight teeth spur gears, having 20^0 involutes full depth teeth is to transmit 12 kW at 300 rpm of the pinion. The speed ration is 3 : 1. The allowable static stress for a gear of CI and pinion of steel are 60 MPa and 105 MPa respectively. Assume number of teeth of pinion = 16 ; face width = 14 times the module ; velocity $(Cv) = \frac{4.58}{4.58+v}$, v being the pitch line velocity in m/s and tooth form factor $(y) = 0.154 - \frac{0.912}{No.of\ teeth}$. design the spur gear and check the gear for wear given $\sigma_{es} = 600\ MPa, E_P = 202\ kN/mm^2, E_G = 100\ kN/mm^2$. **(Dec 2011)(Marks 20)**

7. A air compressor is driven by a 20 kW, 1200 rpm motor, trough a pair of $14\frac{1}{2}^0$ involute spur gears. The speed of the compressor is 300 rpm and the centre distance is approximately 400 rpm. The pinion is to be of cast steel, heat treated and the gear is of CI, grade 35. Assume the gears are subjected to medium shock and working 8 – 10 hrs/ day. Design the gears and check the design for wear load. **(Dec. 2010)(Marks 14)**

8. A gear drive is required to transmitted a maximum power of 2.25 kW. The velocity ratio is 1 :2 and speed of the pinion is 200 rpm. The approximate center distance between the shafts may be taken as 600 mm. the teeth has 20^0 stub involutes profile. The static stress for the gear material (which is CI) may be taken as 60 MPa and face width as 10 times the module. Design the spur gear and check dynamic and wear loads. The deformation or dynamic factor. May be taken as 80 and the material combination factor for wear as 1.4. **(June / July2011)(Marks 15)**

Design procedure for spur gear

Step 1:- *Selection of materials for gear & pinion Ref(T12.7)(P 186)*

Case (i). Same material for pinion & gear.

Such as cast steel 0.2 % C steel untreated ($\sigma_d = 138.3\ N/mm^2$), or steel C 40 untreated ($\sigma_d = 207\ N/mm^2$)

Case (ii). Two different material for pinion & gear

Note:- It is to noted that while selecting different materials for gear and pinion, the material selected for pinion must be stronger than the material selected for gear in other words the material selected for pinion should posses higher value of allowable static stress σ_d compared to that of gear.

Step 2:- *Identifying the weaker member of gears pair.*

(i). When gear and pinion are made of same material, pinion is weaker member and the design is based on pinion.

(ii). When gear and pinion are made of different materials, the strength factor $(\sigma_d\ y)$ is computed as shown and the member with smaller value of $(\sigma_d\ y)$ is taken as the weaker member and the design is based on that member.

To decide the weaker member among the two, the following table has to be formulated.

Particulars	σ_d	y	$\sigma_d\ y$	Remarks
Pinion	σ_{d1}	y_1	$\sigma_{d1}\ y$	
gear	σ_{d2}	y_2	$\sigma_{d2}\ y$	

From table (T 12.7)(P-186) obtain the allowable static stress σ_{d1} & σ_{d2} for the given material.

For the given pressure angle and tooth form, the number of teeth on gears & lewis form factor 'y' for pinion and gear can be obtained by using the following formulas,

$$y = 0.124 - \frac{0.684}{Z} \qquad for\ 14\frac{1}{2}^0\ Involute\ system. \quad (E-12.17a)(P-163)$$

$$y = 0.154 - \frac{0.912}{Z} \qquad for\ 20^0\ Involute\ system. \quad (E-12.17b)(P-163)$$

$$y = 0.175 - \frac{0.95}{Z} \qquad for\ 20^0\ stub\ teeth\ system. \quad (E-12.17a)(P-163)$$

Note:- if teeth are unknown, it is advisable to select minimum 16 teeth on pinion.

Since the load carrying capacity of the tooth is a function of the product $\sigma_d\ y$. The gear where value of $\sigma_d\ y$ is less is the weaker member.

i.e., if $\sigma_{d_1}\ y_1 < \sigma_{d_2}\ y_2$, $Pinion\ is\ weaker.$

if $\sigma_{d_2}\ y_2 < \sigma_{d_1}\ y_1$, $Gear\ is\ weaker.$

Design should be based on weaker member.

Step 3 :- *(i) Tangential tooth load*

Calculation of module (m) of gears.

$$F_t = \frac{1000\ \times P\ \times C_s}{v} \qquad \dots\dots (E\ 12.20\ a)(P\ )$$

Where, P = Power in kW.

$C_s = Service\ factor\ form\ From\ (T\ 12.8)(P-187)$

$[\ generaly\ 1.25\ to\ 1.5\ is\ used\ ]$

$v = Pitch\ line\ velocity\ of\ weaker\ member,\ {}^{m}/_{s}$

$$v = \frac{\pi\, d\, n}{60000} = \frac{\pi\,(m\,Z)\,n}{60000}\ ,\ {}^{m}/_{s} \quad (\therefore d = m\,Z\,)$$

(ii) Tangential tooth load from lewis equation:

$$F_t = \sigma_d\, C_v\, b\, y\, p \qquad \ldots\ldots(E\ 12.15)(P\ 163\,)$$

Where, $\sigma_d =$ $allowable\ static\ stress\ of\ the\ weaker\ member\ (T\ 12.7)$

b = face width 9.5m ≤ b ≤ 12.5 m $\ldots\ldots(E\ 12.18)(P\ 164\,)$

y = Lewis form factor of the weaker member $\ldots\ldots(T\ 12.5)(P\ 184\,)$

$$p = \pi\, m = \frac{\pi\ d}{z}$$

$$C_v = Velocity\ factor\ [\,Barth's formula\,]$$

$$C_v = \frac{3.05}{3.05+v} \qquad for\ v \le 8\ {}^{m}/_{s} \qquad \ldots\ldots(E\ 12.19\ a)(P\ 164\,)$$

$$C_v = \frac{4.58}{4.58+v} \qquad for\ v \le 13\ {}^{m}/_{s} \qquad \ldots\ldots(E\ 12.19\ b)(P\ 164\,)$$

$$C_v = \frac{6.1}{6.1+v} \qquad for\ v \le 6\ {}^{m}/_{s}\ \ to\ 20\ {}^{m}/_{s} \qquad \ldots\ldots(E\ 12.19\ c)(P\ 164\,)$$

$$C_v = \frac{5.55}{5.55+v} \qquad for\ v\ \text{over}\ 20\ {}^{m}/_{s} \qquad \ldots\ldots(E\ 12.19\ d)(P\ 164\,)$$

Note:- Since C_v is based on velocity 'v' which in turn depends on diameter 'd'. Its value can be calculated only if diameter is known.

If diameter is unknown is unknown, velocity 'v' and hence C_v in the equation of module.

Governing equation of module

Case (i) *when diameter of gear & pinion are known or the center distance B/w gears is known*

In the modified lewis equation used to calculate module

$$F_t = \pi\, \sigma_d\, C_v\, b\, y\, p\, m$$

i.e., $m = \frac{F_t}{\pi\, \sigma_d\, C_v\, b\, y}$ $\ldots\ldots(E\ 12.15)(P\ 163\,)$

$$Retain\ m = m\ (an\ unknown\ parameter)$$

$$F_t = \frac{1000\ \times P\ \times C_s}{v} \qquad \ldots\ldots(E\ 12.20\ a)(P - 164\,)$$

$$it\ can\ be\ calculated\ since\quad 'd'\ is\ known$$

Since diameter are known, velocity 'v' and hence C_v, can be computed based on velocity range.

'y', the lewis form factor is retained as 'y' since the number of teeth are unknown.

Note:- when diameter of gears are known, the designer is to compute the number of teeth 'Z' on gear & hence 'y' is retained as 'y'.

After substitution in lewis equation, we obtain

$$m^2 y\,]\ allowable = [\ A\ known\ numerical\ value]$$

And this equation is taken as the governing equation.

Cases (ii) *Diameter of gears are unknown*

In the modified lewis equation used to calculate module

$$F_t = \pi\,\sigma_d\,C_v\,b\,y\,p\,m$$

i.e., $m = \frac{F_t}{\pi\,\sigma_d\,C_v\,b\,y}$ $\quad\ldots\ldots(E\ 12.15)(P\ 163\ )$

$$Retain\ m = m\ (\ an\ unknown\ parameter\ )$$

$$F_t = \frac{1000\ \times P\ \times C_s}{v} \qquad \ldots\ldots(E\ 12.20\ a)(P - 164\ )$$

$$since\quad 'd'\ is\ unknown\ \ (Since\ d = m\,z)$$

Therefore, F_t $will\ be\ in\ term\ of\quad 'm'\ \ i.e.,\ F_t = \cdots\ldots\ldots\ldots\ldots\frac{1}{m}$

σ_d $is\ selected\ from\ table\ of\ material\ \ (T\ 12.7)(\ P\ 186)$

C_v $the\ velocity\ factor\ which\ depends\ on\ velocity\quad 'v'$

$'v'\ connot\ be\ computed\ since\ diameters\ \ are\ unknown.$

$hence\ \ C_v\ is\ retained\ as\ \ C_v\ \ in\ the\ module\ equation.$

b = face width is taken as $9.5\mathrm{m} \le \mathrm{b} \le 12.5\ \mathrm{m}$ $\ldots(E\ 12.18)(P\ 164\ )$

$the\ lewis\ form\ factor\ is\ calculated\ as\ assuming\ suitable$

$number\ of\ teeth\ on\ weaker\ member\ with\ \ minimum.$

16 teeth on pinion with the substitution of the above values.

After substitution in lewis equation, we obtain the controlling equation as governing equation for module reduced to

$$m^3\ C_v\ ]\ allowable = [\ A\ known\ numerical\ value]$$

Is used to determine 'm'. the allowable module by selecting a standard preferred module table (T 12.2)(P 182)

Trails to find 'm'

Note:- Continue the trails with increasing or decreasing values of standard module till the quantity LHS equals the numerical values of the RHS of governing equation of module or less than the numerical values of the RHS of equation.

Dimensions

Calculate all important geometric parameter of tooth profile by using the equations given in Table (T 12.4)(P 183)

Properties of involutes teeth

Tooth Characteristics		$14\frac{1}{2}^0$ $Involute\ system$	20^0 $Involute\ system$	20^0 $stub\ teeth\ \ system.$
Pressure angle	θ	$14\frac{1}{2}^0$	20^0	20^0
Addendum	h_a	m	m	0.8 m
Dedendum	h_f	1.57 m	1.57 m	m
Thickness of the tooth	t	1.571 m	1.571 m	1.571 m
Teeth	z			
Pitch circle diameter	d			

Check for the stress

Calculate the induced stress by the equation

$$\sigma_{ind} = (\sigma_d\,C_v)_{ind} = \frac{F_t}{b\,y\,p} \qquad \ldots\ldots(E\ 12.15)(P\ 163\ )$$

Calculate the allowable stress by the equation

$$\sigma_{allow} = (\sigma_d\,C_v)_{allow}$$

If $\sigma_{ind} < \sigma_{allow}$, then the design is satisfactory.

Checking for dynamic load

Acc to bucking ham's equation

Dynamic load $F_d = F_t + \frac{K_3\, v\,(C\, b + F_t)}{K_3\, v + \sqrt{(C\, b + F_t)}}$ $\ldots\ldots (E\ 12.33)(P\ 166\ )$

Where C = load factor, using table (T 12.12) the load factor C can obtained

If the class of gear is known then the error 'e' can be obtained from (T 12.13)(Fig 12.3)(P 205)

(Explained in detail in numerical example)

Endurance strength of the gear tooth

$$F_{en} = \sigma_{en}\, b\, Y\ m \qquad \ldots\ldots (E\ 12.34)(P\ 166\ )$$

Where $Y = \pi\, y$, $y = Lewis\ from\ factor\ of\ the\ weaker\ member$.

For safer design must be less than the allowable endurance strength $(F_d < F_{en})$

Note:- Allowable static stress (σ_d) used in the lewis equation is $1/3$ of ultimate stress and $\sigma_{en} = 0.5\, \sigma_u$.

Check for wear load

Acc to bucking ham's equation wear load

$$F_w = d_1\, b\, Q\ k \qquad \ldots\ldots (E\ 12.36\ a)(P\ 167\ )$$

Where ratio factor $Q = \frac{2\, d_2}{d_2 + d_1} = \frac{2\, z_2}{z_2 + z_1}$ $\ldots\ldots (E\ 12.36\ c)(P\ 167\ )$

Load stress factor $k = \frac{\sigma_{es}^2\, Sin \propto}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$ $\ldots\ldots (E\ 12.36\ b)(P\ 167\ )$

From Table (T 12.16 & 12.17)

Surface endurance limit

$$\sigma_{es} = (2.75\, BHN - 70) \quad MPa \qquad \ldots\ldots (E\ 12.36\ d)(P\ 167\ )$$

$$BHN = Brinell\ hardness\ Number$$

Select core hardness no for given material and value of σ_{en} from Table (T-12.15)(P-192)

For safer design F_w must be greater than F_d , $(F_d \leq F_w)$

If $F_w > F_d$ then make one or more following changes

(i). calculate the dynamic load (F_d) by decreasing the tooth error 'e'

(ii). Decrease the module 'm'

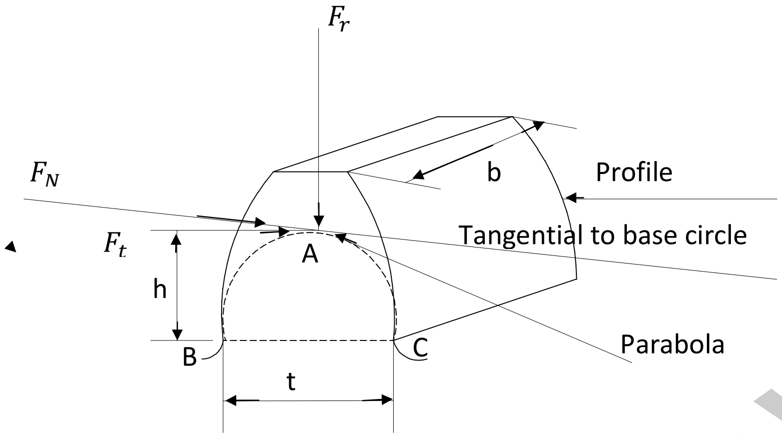
(iii). Increase the face width 'b'

(iV). Increase the surface hardness.

BEAM STRENGTH OF SPUR GEAR TEETH OR LEWIS EQUATION

The beam strength of gear teeth is determined from an equation known as lewis equation. The load carrying ability of the gears as determined by this equation given satisfactory results. Lewis assumed that the load is being transmitted from one gear to another; it is all given and taken by one tooth. When contact begins, the load is assumed to be the end of the driven teeth and as contact ceases, it is at the end of driving teeth.

Consider each tooth as a cantilever beam loaded by a normal load F_N as shown in below Figure. It is resolved into two components (i) tangential component (F_t) (ii) Radial component (F_r)



The tangential component F_t induces a bending stress which tends to break the tooth. The radial component F_r induces compressive stress of relatively small magnitude therefore its effect on the tooth may be neglected. Hence the bending stress is used as the basis for design calculations.

The critical section may be obtained by drawing a parabola through A and tangential to the curve at B and C. this parabola shown in dotted line outlines a

beam of uniform strength. But since the tooth is larger than the parabola at every section except BC, it concludes BC is the critical section.

The maximum value of bending stress at BC is given by, $\sigma = \frac{M}{I} . c$ where

$$M = maximum\ bending\ moment\ at\ section\ BC = F_t\ x\ h$$

$$c = \frac{t}{2}, t = thickness\ of\ tooth$$

$$I = \frac{b\ t^3}{12}, b = face\ width\ of\ tooth$$

$$\therefore \sigma = \frac{(F_t \times h)\left(\frac{t}{2}\right)}{\left(\frac{b\ t^3}{12}\right)} = \frac{(F_t\ x\ h)\ 6}{b\ t^2}$$

$$\therefore F_t = \sigma \times b \times \frac{t^2}{6\ h}$$

Since t and h are variables which depends upon the shape of tooth and circular pitch, the quantity $\frac{t^2}{6\ h}$ may be replaced by 'p y' where y is known as lewis form factor or tooth form factor. Therefore the final form of strength formula is $F_t = \sigma\ b\ y\ p$. This equation is called lewis equation or beam strength of tooth. The value of 'y' for various tooth system are

$$y = 0.124 - \frac{0.684}{Z} \quad for\ 14\frac{1}{2}^{0}\ Involute\ system.\ (E - 12.17a)(P - 163)$$

$$y = 0.154 - \frac{0.912}{Z} \quad for\ 20^0\ Involute\ system.\ (E - 12.17b)(P - 163)$$

$$y = 0.175 - \frac{0.95}{Z} \quad for\ 20^0\ stub\ teeth\ system.\ (E - 12.17a)(P - 163)$$

The permissible stress in lewis equation depends upon materials, pitch line velocity and load condition, according to Barth formula the permissible stress

$$\sigma = \sigma_0 \times C_v , \qquad C_v = velocity\ factor \quad and$$

$\sigma_0 \; = allowable\ static\ stress\ from\ Table\ (T - 12.7)$

$$C_v = \frac{3.05}{3.05 + v} \quad for\ v \leq 8\ {}^{m}/_{s} \quad \ldots\ldots(E\ 12.19\ a)(P\ 164)$$

$$C_v = \frac{4.58}{4.58+v} \quad for\ v \leq 13\ {}^{m}/_{s} \quad \ldots\ldots(E\ 12.19\ b)(P\ 164)$$

$$C_v = \frac{6.1}{6.1+v} \quad for\ v \leq 6\ {}^{m}/_{s}\ \ to\ 20\ {}^{m}/_{s} \quad \ldots\ldots(E\ 12.19\ c)(P\ 164)$$

$$C_v = \frac{5.55}{5.55+v} \quad for\ v\ \text{ over } 20\ {}^{m}/_{s} \quad \ldots\ldots(E\ 12.19\ d)(P\ 164)$$

$$v = \frac{\pi\ d\ n}{60000} = \frac{\pi\ (m\ Z)\ n}{60000}\ ,\ {}^{m}/_{s} \quad (\therefore d = m\ Z)$$

Therefore the final form of lewis equation for tangential tooth load in terms of circular pitch $F_t = \sigma_d\ C_v\ b\ y\ p$

1. A pair of carefully cut gears with 20^0 FDI profile is used to transmit 12 kW at 1200 rpm of pinion. The gear has to rotate at 300 rpm. The material used for both pinion and gear is medium carbon steel whose allowable bending stresses may be taken as 230 MPa. Determine the module and face width of spur pinion and gear. Check the pair for wear strength against dynamic load. Take 24 teeth on pinion. Assume modulus of elasticity as 202 GPa. **(Dec.09/Jan. 10)(Marks 15)**

Data: $\propto = 20^0\ FDI\ ; P = 12\ kW\ ;\ n_1 = 1200\ rpm\ ;\ \ n_2 = 300\ rpm\ ;\ \sigma_{d_1} = \sigma_{d_2} = 230\ MPa\ ; z_1 = 24\ teeth\ ;\ E_1 = E_2 = 202\ GPa\ ;$

$carefully\ cut\ spur\ gear.$

Solution: *Note: - Since the diameters are unknown, it is possible to design for smallest pitch diameter.*

Velocity ratio (i)

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_1}{d_2}$$

$$i = \frac{1200}{300} = 4$$

$$\therefore \text{ Velocity ratio} \qquad i = 4$$

$$\therefore\ \ z_2 = i\ \ z_1 = 4\ \times 24 = 96\ Teeth$$

To determine 'y' the lewis form factor

For 20^0 FDI profile

$$y = 0.154 - \frac{0.912}{Z} \quad for\ 20^0\ Involute\ system.$$

$$(E - 12.17b)(\ P - 163) \quad (General\ equation)$$

$$For\ Pinion, \quad y_1 = 0.154 - \frac{0.912}{Z_1}$$

$$y_1 = 0.154 - \frac{0.912}{24}$$

$$y_1 = 0.116$$

$$For\ Gear, \quad y_2 = 0.154 - \frac{0.912}{Z_2}$$

$$y_2 = 0.154 - \frac{0.912}{96}$$

$$y_2 = 0.1445$$

Identifying the weaker member

Particulars	σ_d	y	$\sigma_d\ y$	Remarks
Pinion	230	0.116	26.68	**Weaker**
gear	230	0.1445	33.235	

Since $\sigma_{d_1}\ y_1 < \sigma_{d_2}\ y_2$, *Pinion is weaker member.*

Design should be based on Pinion (weaker member).

Tangential tooth load(F_t)

$$F_t = \frac{1000 \times P \times C_s}{v} \quad \ldots\ldots (E\ 12.20\ a)(P-164) \quad (General\ equation)$$

$$F_{t_1} = \frac{1000 \times P \times C_s}{v_1} \quad (Design\ equation)$$

Where, P = 12 kW= Power in kW.

$$C_s = Service\ factor\ form\ From\ (T\ 12.8)(P-187)$$

Assume medium shock and $8-10$ *hours duty per day*

$C_s = 1.5$; $Speed = n_1 = 1200\ rpm$

$$v = Pitch\ line\ velocity\ of\ weaker\ member, \quad m/s$$

$$v = \frac{\pi\ d\ n}{60000} = \frac{\pi\ (m\ Z)\ n}{60000}, \quad m/s \quad (\because d = m\ Z) \quad (General\ equation)$$

$$v_1 = \frac{\pi\ d_1\ n_1}{60000} = \frac{\pi\ (m\ Z_1)\ n_1}{60000} = \frac{\pi \times (m \times 24) \times 1200}{60000}$$

$$v_1 = 1.508\ m \quad m/s$$

$$\therefore \quad F_{t_1} = \frac{1000 \times 12 \times 1.5}{1.508\ m}$$

$$F_{t_1} = \frac{11936.34}{m} \quad N \quad \ldots\ldots\ldots Equn\ (i)$$

(ii) Tangential tooth load from lewis equation:

$$F_t = \sigma_d\ C_v\ b\ y\ p \quad \ldots\ldots (E\ 12.15)(P\ 163) \quad (General\ equation)$$

$$F_{t_1} = \sigma_{d_1}\ C_{v_1}\ b\ y_1\ p \quad (Design\ equation)$$

Where, $\sigma_{d_1} =$ *allowable static stress of the pinion* $= 230\ MPa$

b = face width 9.5m ≤ b ≤ 12.5 m $\ldots\ldots (E\ 12.18)(P\ 164)$

Take face width, b = 10 m

y_1 = Lewis form factor of the Pinion

$$y_1 = 0.116$$

$$Circular\ Pitch, \quad p = \pi\ m$$

$$F_{t_1} = 230 \times C_{v_1} \times (10 \times m) \times 0.116 \times (\pi \times m)$$

$$F_{t_1} = 838.177\ m^2\ C_{v_1} \quad N \quad \ldots\ldots\ldots Equn\ (ii)$$

Equating equation (*i*) & (*ii*)

$$838.177\, m^2\, C_{v_1} = \frac{11936.34}{m}$$

$$m^3\, C_{v_1} = 14.241 \qquad \ldots\ldots\ldots\ Equn\ (iii)$$

To find module 'm'

Trial 1:- *Let select 'm = 3'*

$$Velocity\ \ v_1 = 1.508\, m = 1.508\ \times 3$$

$$v_1 = 4.524 \quad {}^{m}/_{s}$$

$$Velocity\ factor\ [\ Barth'sformula\ ]\ \ (\ C_v)$$

$$C_v = \frac{3.05}{3.05 + v} \quad for\ v \le 8\ {}^{m}/_{s} \quad \ldots\ldots (E\ 12.19\ a)(P\ 164\ )\ (General\ equation)$$

$$C_{v_1} = \frac{3.05}{3.05 + v_1} \qquad (Design\ equation)$$

$$C_{v_1} = \frac{3.05}{3.05 + 4.524}$$

$$C_{v_1} = 0.4027$$

Now from equation ….. (*iii)*

$$m^3\, C_{v_1} \ge 14.241 \qquad \ldots\ldots\ldots\ Equn\ (iii)$$

$$3^3 \times 0.4027\ = 14.241$$

$$10.873\ < 14.241 \qquad \therefore not\ suitable.$$

Trial 2:- *Let select 'm = 4'*

$$Velocity\ \ v_1 = 1.508\, m = 1.508\ \times 4$$

$$v_1 = 6.032 \quad {}^{m}/_{s}$$

$$Velocity\ factor\ [\ Barth'sformula\ ]\ \ (\ C_v)$$

$$C_{v_1} = \frac{3.05}{3.05 + v_1} \qquad (Design\ equation)$$

$$C_{v_1} = \frac{3.05}{3.05 + 6.032}$$

$$C_{v_1} = 0.3358$$

Now from equation ….. (*iii)*

$$m^3\, C_{v_1} \ge 14.241 \qquad \ldots\ldots\ldots\ Equn\ (iii)$$

$$4^3 \times 0.3358\ = 14.241$$

$$21.49\ > 14.241 \qquad \therefore suitable.$$

$$\therefore Take \qquad m = 4\, mm$$

∴ Face width 'b' b = 10 m = 10 x 4 = 40 mm.

$$\therefore \quad F_{t_1} = \frac{11936.34}{m} = \frac{11936.34}{4} = 2984.085\ N$$

Check for the stress

Allowable stress by the equation

$$\sigma_{allow} = \ (\sigma_d\, C_v)_{allow} \qquad (General\ equation)$$

$$\sigma_{allow} = \ \left(\sigma_{d_1}\, C_{v_1}\right)_{allow} \qquad (Design\ equation)$$

$$\sigma_{allow} = 230\ \times 0.3358$$

$$\sigma_{allow} = 77.234\ MPa$$

Induced stress

$$\sigma_{ind} = (\sigma_d\, C_v)_{ind} = \frac{F_t}{b\, y\, p} \quad \ldots\ldots (E\ 12.15)(P\ 163\ ) \quad (General\ equation)$$

$$\sigma_{ind} = (\sigma_d\, C_v)_{ind} = \frac{F_{t_1}}{b\, y_1\, p} \qquad (Design\ equation)$$

$$\sigma_{ind} = \frac{2984.085}{40\ \times 0.116\ \times\ \pi \times 4}$$

$$\sigma_{ind} = 51.178\ MPa$$

If $\sigma_{ind}\ <\ \sigma_{allow}\ (51.178\ MPa < 77.234\ MPa)$, then the design is satisfactory.

Dimensions

Calculate all important geometric parameter of tooth profile by using the equations given in Table (T 12.4)(P 183)

Properties of involutes teeth

Tooth Characteristics		20^0 FDI	*Pinion*	*Gear*
Addendum	h_a	m	4	4
Dedendum	h_f	1.57 m	6.28	6.28
Thickness of the tooth	t	1.571 m	6.284	6.284
No of Teeth	z		24	96
Pitch circle diameter	d	m z	96	384

Checking for dynamic load

Acc to bucking ham's equation

Dynamic load(F_d)

$$F_d = F_t + \frac{K_3\, v\,(C\, b\ +\ F_t)}{K_3\, v\ + \sqrt{(C\, b\ +\ F_t)}} \quad \ldots\ldots (E\ 12.33)(P\ 166\ ) \qquad (General\ equation)$$

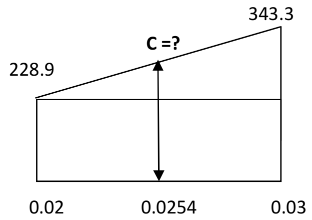
$$F_d = F_{t_1} + \frac{K_3\, v_1\,(C\, b\ +\ F_{t_1})}{K_3\, v_1\ + \sqrt{(C\, b\ +\ F_{t_1})}} \qquad (Design\ equation)$$

From Table T(12.13) (P- 191) for carefully cut gears for m = 4 mm

Machining error, e= 0.0254 mm

From T(12.12) (P- 190) for e = 0.0254 mm $\alpha = 20^0\ FDI$ and steel pinion – steel gear

Error 'e'	C
0.02	228.9
0.0254	?
0.03	343.3



0.02	228.9
0.0254	C

$$C\ \times 0.02 = 228.9\ \times 0.0254$$

$$C = \frac{228.9\ \times 0.0254}{0.02}$$

$$C = 290\ MPa$$

Where, $F_{t_1} = 2984.085\ N\ ;\ K_3 = 20.67\ ;\ v_1 = 6.032\ {}^{m}/_{s}\ ;$

$$\mathrm{C\ =\ 290\ MPa;\ b =\ 40\ mm;}$$

$$F_d = 2984.085 + \frac{20.67 \times 6.032 \times (290 \times 40 + 2984.085)}{20.67 \times 6.032 + \sqrt{(290 \times 40 + 2984.085)}}$$

$$F_d = 2984.085 + \frac{1818364.72}{245.45}$$

$$F_d = 2984.085 + 7408.29$$

$$F_d = 10392.4 \ N$$

Check for endurance strength $(\boldsymbol{F_{en}})$

$$F_{en} = \sigma_{en} b \ Y \ m \quad \ldots\ldots (E\ 12.34)(P\ 166) \quad (General\ equation)$$

$$F_{en_1} = \sigma_{en_1} b \ Y_1 \ m \quad (Design\ \ equation)$$

In lewis equation the allowable bending stress is approximately $\frac{1}{3}$ of ultimate stress.

$\therefore$ Ultimate stress of the pinion material(σ_u),

$$\sigma_u = 3 \times \ \sigma_{d_1} = 3 \times \ 230 = 690 \ MPa$$

Now endurance limit $\sigma_{en} = 0.5 \times \ \sigma_u = 0.5 \times \ 690 = 345 \ MPa$

Where, $\sigma_{en} = 345 \ MPa$; $b = 40 \ mm$; $Y_1 = \ \pi \ y_1 = \pi \ \times 0.116 = 0.3644 \ {}^{m}/_{s}$; m = 4 mm.

$$F_{en_1} = 345 \ \times 40 \times 0.3644 \times 4$$

$$F_{en_1} = 20114.88 \ N$$

Since $F_{en} \ > \ F_d$ $(20114.88 \ N > \ 10392.4 \ N)$, the design will be ***satisfactory from the point of wear or durability.***

Check for wear load (F_w)

Acc to bucking ham's equation wear load

$$F_w = \ d_1 \ b \ Q \ \ k \quad \ldots\ldots (E\ 12.36\ a)(P\ 167) \quad (General\ equation)$$

$$F_w = \ d_1 \ b \ Q \ \ k \quad (Design\ equation)$$

Where ratio factor $Q = \frac{2\,d_2}{d_2 + \,d_1} = \frac{2\,z_2}{z_2 + \,z_1}$ $\ldots\ldots (E\ 12.36\ c)(P\ 167)$

$$Q = \frac{2\ z_2}{z_2 + \ z_1} = \frac{2 \ \times 96}{96 \ + \ 24}$$

$$Q = 1.6$$

From table (T-12.15)(P-192) for $\sigma_{en} = 345 \ MPa$ Core(BHN)=200
BHN $(Brinell\ hardness\ Number)$

Surface endurance limit

$$\sigma_{es} = \ (2.75 \ BHN - 70) \quad MPa \quad \ldots\ldots (E\ 12.36\ d)(P\ 167)$$

$$\sigma_{es} = \ (2.75 \ \times 200 \ - 70)$$

$$\sigma_{es} = 480 \ MPa$$

Load stress factor $k = \frac{\sigma_{es}{}^2 \ Sin \propto}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$ $\ldots\ldots (E\ 12.36\ b)(P\ 167)$

$$k = \frac{480^2 \ Sin\ 20}{1.4}\left[\frac{1}{2.02 \times 10^5} + \frac{1}{2.02 \times 10^5}\right]$$

$$k = 0.5573 \quad MPa$$

i.e., $F_w = \ d_1 \ b \ Q \ \ k = 96 \times 40 \times 1.6 \times 0.5573$

$$F_w = 3424.05 \ N$$

Since $F_w \ < \ F_d$ $(3424.05 \ N < \ 10392.4 \ )$, the design will be **unsatisfactory from the point of wear or durability.**

For safer design

$$F_w \geq F_d$$

$$i.e., d_1\, b\, Q\;\; k \;\geq F_d$$

$$i.e., 96 \times 40 \times 1.6 \times k \geq 10392.4$$

$$\therefore Load\ stress\ factor\;\; k \geq 1.70$$

$$i.e., \frac{{\sigma_{es}}^2\ Sin \propto}{1.4}\left[\frac{1}{E_1}+\frac{1}{E_2}\right] \geq 1.70$$

$$i.e., \frac{{\sigma_{es}}^2\ Sin\ 20}{1.4}\left[\frac{1}{2.1\times10^5}+\frac{1}{2.1\times10^5}\right] \geq 1.70$$

$$\therefore\;\; {\sigma_{es}}^2 \geq 730658.72$$

$$\therefore\;\; \sigma_{es} \geq 854.8\ MPa$$

$$i.e., (2.75\ \times BHN\ - 70) \geq 854.8$$

$$i.e., BHN \geq 336$$

OR

From the Table (T 12.16)(P- 193) for $\alpha = 20^0$

$$k \geq 1.70\ MPa$$

Surface hardness no for pinion = 400 BHN

Surface hardness no for gear = 300 BHN

Hence suggested average surface hardness for the gear pair $\geq 350\ BHN$

Hence if the average surface hardness of the gear pair is more than 350 BHN, then the design will be satisfactory from the stand point of durability or wear also.

2. A pair of spur gears has to transmit 20 kW from a shaft rotating at 1000 rpm to a parallel shaft which is to rotate at 310 rpm. Number of teeth on pinion is 31 with 20^0 full depth involute tooth form. The material for pinion is steel C40 untreated with allowable static stress 207 MPa and the material for gear is cast steel 0.20% C untreated with allowable static stress 138.3 MPa. Determine the module and face width of the gear pair. Also find dynamic tooth load on the gears. Take the service factor as 1.5 and check the wear resistance of the gear.

(May/June 2010)(Marks 20)

Data: $\propto = 20^0\ FDI$; $P = 20\ kW$; $n_1 = 1000\ rpm$; $n_2 = 310\ rpm$; $z_1 = 31\ teeth$; $C_s = 1.5$.

$$pinion\ material - Steel\ C\ 40\ untreated\ ;$$

$$gear\ material - Steel\ 0.20\%\ untreated\ ;$$

From table (T-12.7)(P-186) allowable stress for

$$pinion\ material - Steel\ C\ 40\ untreated; \quad \sigma_{d_1} = 207\ MPa$$

$$gear\ material - Steel\ 0.20\%\ untreated; \quad \sigma_{d_2} = 138.3\ MPa$$

Solution: *Note:* - Since the diameters are unknown, it is possible to design for smallest pitch diameter.

Velocity ratio (i)

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_1}{d_2}$$

$$i = \frac{1000}{310} = 3.2258$$

$$\therefore \text{ Velocity ratio} \qquad i = 3.2258$$

$$\therefore \quad z_2 = i \ z_1 = 3.2258 \times 31 = 100 \, Teeth$$

To determine 'y' the lewis form factor

For 20⁰ FDI profile

$$y = 0.154 - \frac{0.912}{Z} \quad for\ 20^0\ FDI\ system.$$

$$(E - 12.17b)(\ P - 163) \quad (General\ equation)$$

$$For\ Pinion\ y_1 = 0.154 - \frac{0.912}{Z_1}$$

$$y_1 = 0.154 - \frac{0.912}{31}$$

$$y_1 = 0.1246$$

$$For\ Gear \quad y_2 = 0.154 - \frac{0.912}{Z_2}$$

$$y_2 = 0.154 - \frac{0.912}{100}$$

$$y_2 = 0.14488$$

Identifying the weaker member

Particulars	σ_d	y	$\sigma_d\ y$	Remarks
Pinion	207	0.1246	25.8	
Gear	138.3	0.14488	20.037	**Weaker**

Since $\sigma_{d_2}\ y_2 < \sigma_{d_1}\ y_1$, $Gear\ is\ weaker\ member.$

Design should be based on Gear (weaker member).

Tangential tooth load

$$F_t = \frac{1000 \times P \times C_s}{v} \quad \dots \dots (E\ 12.20\ a)(P - 164\ ) \quad (General\ equation)$$

$$F_{t_2} = \frac{1000 \times P \times C_{s_2}}{v_2} \quad (Design\ equation)$$

Where, P = 20 kW= Power in kW.

$$C_s = Service\ factor\ form\ \ From\ (T\ 12.8)(P - 187)$$

$Assume\ medium\ shock\ and\ 8 - 10\ hours\ duty\ per\ day$

$$C_{s_2} = 1.5 \ ;\ Speed = \ n_2 = 310\ rpm$$

$$v = Pitch\ line\ velocity\ of\ weaker\ member, \quad m/s$$

$$v = \frac{\pi\, d\, n}{60000} = \frac{\pi\ (m\ Z)\ n}{60000}, \quad m/s\ \ (\therefore d = m\ Z\ ) \quad (General\ equation)$$

$$v_2 = \frac{\pi\ d_2\ n_2}{60000} = \frac{\pi\ (m\ Z_2)\ n_2}{60000} = \frac{\pi \times (m \times 100) \times 310}{60000}$$

$$v_2 = 1.623\ m \quad m/s$$

$$\therefore \quad F_{t_2} = \frac{1000 \times 20 \times 1.5}{1.623\ m}$$

$$F_{t_2} = \frac{18484.3}{m} \quad N \quad \dots \dots \dots Equn\ (i)$$

(ii) Tangential tooth load from lewis equation:

$$F_t = \ \sigma_d\ C_v\ b\ y\ p \quad \dots \dots (E\ 12.15)(P\ 163\ ) \quad (General\ equation)$$

$$F_{t_2} = \ \sigma_{d_2}\ C_{v_2}\ b\ y_2\ p \quad (Design\ equation)$$

Where $\sigma_{d_2} = \ allowable\ static\ stress\ of\ the\ pinion \ = 138.3\ MPa$

$$b = \text{face width} \quad 9.5\text{m} \le b \le 12.5\text{ m} \quad \ldots\ldots(E\ 12.18)(P\ 164)$$

$$\text{Take face width,} \quad b = 10\text{ m}$$

$$y_2 = \text{Lewis form factor of the Pinion}$$

$$y_2 = 0.14488$$

$$Circular\ Pitch, \quad p = \pi\ m$$

$$F_{t_2} = 138.3 \times C_{v_2} \times (10 \times m) \times 0.14488 \times (\pi \times m)$$

$$F_{t_2} = 629.48\ m^2\ C_{v_2} \quad N \quad \ldots\ldots\ldots\ Equn\ (ii)$$

Equating equation (*i*) & (*ii*)

$$629.48\ m^2\ C_{v_2} = \frac{18484.3}{m}$$

$$m^3\ C_{v_2} \ge 29.36 \quad \ldots\ldots\ldots\ Equn\ (iii)$$

To find module 'm'

Trial 1:- *Let select ' m = 4'*

$$Velocity \quad v_2 = 1.623\ m = 1.623 \times 4$$

$$v_2 = 6.492 \quad {}^{m}/_{s}$$

$$Velocity\ factor\ [\ Barth's formula\ ]\ (\ C_v)$$

$$C_v = \frac{3.05}{3.05 + v} \quad for\ v \le 8\ {}^{m}/_{s}$$

$$\ldots\ldots(E\ 12.19\ a)(P\ 164)\ (General\ equation)$$

$$C_{v_2} = \frac{3.05}{3.05 + v_2} \quad (Design\ equation)$$

$$C_{v_2} = \frac{3.05}{3.05 + 6.492}$$

$$C_{v_2} = 0.3196$$

Now from equation ..... (*iii)*

$$m^3\ C_{v_2} \ge 29.36 \quad \ldots\ldots\ldots\ Equn\ (iii)$$

$$4^3 \times 0.3196 = 29.36$$

$$20.45 < 29.36 \quad \therefore\ not\ suitable.$$

Trial 2:- *Let select ' m = 5'*

$$Velocity \quad v_2 = 1.623\ m = 1.623 \times 5$$

$$v_2 = 8.115 \quad {}^{m}/_{s}$$

$$Velocity\ factor\ [\ Barth's formula\ ]\ (\ C_v)$$

$$C_{v_2} = \frac{4.58}{4.58 + v_2} \quad for\ v \le 13\ {}^{m}/_{s} \quad (Design\ equation)$$

$$C_{v_2} = \frac{4.58}{4.58 + 8.115}$$

$$C_{v_2} = 0.36077$$

Now from equation ..... (*iii)*

$$m^3\ C_{v_2} = 29.36 \quad \ldots\ldots\ldots\ Equn\ (iii)$$

$$5^3 \times 0.36077 = 29.36$$

$$45.096 > 29326 \quad \therefore\ suitable.$$

$$\therefore Take \quad m = 5\ mm$$

∴ Face width 'b' b = 10 m = 10 x 5 = 50 mm.

$$\therefore \quad F_{t_2} = \frac{18484.3}{m} = \frac{18484.3}{5} = 3696.86\ N$$

Dimensions

Calculate all important geometric parameter of tooth profile by using the equations given in Table (T 12.4)(P 183)

Properties of involutes teeth

Tooth Characteristics		20^0 FDI	$Pinion$	$Gear$
Addendum	h_a	m	5	5
Dedendum	h_f	1.57 m	7.85	7.85
Thickness of the tooth	t	1.571 m	7.855	7.855
No of Teeth	z		31	100
Pitch circle diameter	d	m z	155	500

Check for the stress

Allowable stress by the equation

$$\sigma_{allow} = (\sigma_d \, C_v)_{allow} \qquad (General\ equation)$$

$$\sigma_{allow} = \left(\sigma_{d_2} \, C_{v_2}\right)_{allow} \qquad (Design\ equation)$$

$$\sigma_{allow} = 138.3 \times 0.361$$

$$\sigma_{allow} = 50\ MPa$$

induced stress

$$\sigma_{ind} = (\sigma_d \, C_v)_{ind} = \frac{F_t}{b\, y\, p}$$

$$\ldots\ldots(E\ 12.15)(P\ 163\ ) \qquad (General\ equation)$$

$$\sigma_{ind} = (\sigma_d \, C_v)_{ind} = \frac{F_{t_2}}{b\, y_2\, p} \qquad (Design\ equation)$$

$$\sigma_{ind} = \frac{3696.86}{50 \times 0.14488 \times \pi \times 5}$$

$$\sigma_{ind} = 32.49\ MPa$$

If $\sigma_{ind} < \sigma_{allow} (32.49\ MPa < 50\ MPa)$, then the design is satisfactory.

Checking for dynamic load

Acc to bucking ham's equation

Dynamic load

$$F_d = F_t + \frac{K_3\, v\, (C\, b + F_t)}{K_3\, v + \sqrt{(C\, b + F_t)}} \quad \ldots\ldots(E\ 12.33)(P\ 166\ ) \ (General\ equation)$$

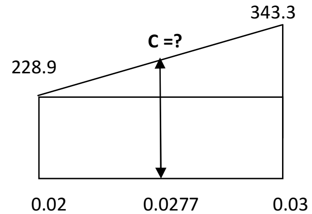
$$F_d = F_{t_2} + \frac{K_3\, v_2\, \left(C\, b + F_{t_2}\right)}{K_3\, v_2 + \sqrt{\left(C\, b + F_{t_2}\right)}} \qquad (Design\ equation)$$

From Table T(12.13) (P- 191) for carefully cut gears for m = 5 mm

Machining error, e= 0.0277 mm

From T(12.12) (P- 190) for e = 0.0277 mm $\alpha = 20^0\ FDI$ and steel pinion – steel gear

Error 'e'	C
0.02	228.9
0.0254	?
0.03	343.3



0.02	228.9
0.0277	C

$$C \times 0.02 = 228.9 \times 0.0277$$

$$C = \frac{228.9 \times 0.0277}{0.02}$$

$$C = 317\ MPa$$

Where, $F_{t_2} = 3696.86\ N$; $K_3 = 20.67$; $v_2 = 8.115\ \ {}^{m}/_{s}$;

$$\mathrm{C} = 317\ \mathrm{MPa};\ \mathrm{b} = 50\ \mathrm{mm};$$

$$F_d = 3696.86 + \frac{20.67 \times 8.115 \times (317 \times 50 + 3696.86)}{20.67 \times 8.115 + \sqrt{(317 \times 50 + 3696.86)}}$$

$$F_d = 3696.86 + \frac{3278732.633}{307.55}$$

$$F_d = 3696.86 + 10660.8$$

$$F_d = 14357.66\ N$$

Check for endurance strength(F_{en})

$$F_{en} = \sigma_{en} b\ Y\ m \qquad \ldots\ldots(E\ 12.34)(P\ 166) \qquad (General\ equation)$$

$$F_{en_1} = \sigma_{en_2} b\ Y_2\ m \qquad (Design\ \ equation)$$

In lewis equation the allowable bending stress is approximately $\frac{1}{3}$ of ultimate stress.

∴ ultimate stress of the pinion material, $\sigma_u = 3 \times \sigma_{d_2} = 3 \times 138.3 = 415\ MPa$

Now endurance limit $\sigma_{en} = 0.5 \times \sigma_u = 0.5 \times 690 = 207.5\ MPa$

Where, $\sigma_{en} = 207.5\ MPa$; $b = 50\ mm$; $Y_2 = \pi\ y_2 = \pi \times 0.14488 = 0.45515\ {}^{m}/_{s}$; m = 5 mm.

$$F_{en_2} = 207.5 \times 50 \times 0.45515 \times 5$$

$$F_{en_2} = 23610.9\ N$$

Since $F_{en} > F_d$ ($23610.9\ N > 14357.86\ N$), the design will be ***satisfactory from the point of wear or durability.***

Check for wear load

Acc to bucking ham's equation wear load

$$F_w = d_1\ b\ Q\ k \qquad \ldots\ldots(E\ 12.36\ a)(P\ 167) \qquad (General\ equation)$$

$$F_w = d_1\ b\ Q\ k \qquad (Design\ equation)$$

Where ratio factor

$$Q = \frac{2\ d_2}{d_2 + d_1} = \frac{2\ z_2}{z_2 + z_1} \qquad \ldots\ldots(E\ 12.36\ c)(P\ 167)$$

$$Q = \frac{2\ z_2}{z_2 + z_1} = \frac{2 \times 100}{100 + 31}$$

$$Q = 1.527$$

From table (T-12.15)(P-192) for $\sigma_{en} = 207.5\ MPa$ Core (BHN) = 120 BHN ($Brinell\ hardness\ Number$)

From table (T-1.1)(P-7) for carbon steel

Young's modulus $E = E_1 = E_2 = 202\ GPa = 202 \times 10^3\ MPa$

Surface endurance limit

$$\sigma_{es} = (2.75\ BHN - 70)\quad MPa \qquad \ldots\ldots(E\ 12.36\ d)(P\ 167)$$

$$\sigma_{es} = (2.75 \times 120 - 70)$$

$$\sigma_{es} = 260\ MPa$$

Load stress factor

$$k = \frac{\sigma_{es}^2 \, Sin \propto}{1.4}\left[\frac{1}{E_1}+\frac{1}{E_2}\right] \quad \ldots\ldots(E\ 12.36\ b)(P\ 167\ )$$

$$k = \frac{260^2\ Sin\ 20}{1.4}\left[\frac{1}{2.02\times 10^5}+\frac{1}{2.02\times 10^5}\right]$$

$$k = 0.1635 \quad MPa$$

i.e., $F_w = d_1\ b\ Q\ \ k = 155\times 50\times 1.527\times 0.1635$

$$F_w = 1934.9\ N$$

Since $F_w < F_d$ $(1934.9\ N < 14348\ N)$, the design will be **unsatisfactory from the point of wear or durability.**

For safer design

$$F_w \ \geq\ F_d$$

$$i.e., d_1\ b\ Q\ \ k \ \geq\ F_d$$

$$155\times 50\times 1.527\times\ k\ \geq\ 14348$$

$$\therefore Load\ stress\ factor\ \ k\ \geq 1.212$$

$$i.e., \frac{\sigma_{es}^2\ Sin\ \propto}{1.4}\left[\frac{1}{E_1}+\frac{1}{E_2}\right] \geq 1.212$$

$$i.e., \frac{\sigma_{es}^2\ Sin\ 20}{1.4}\left[\frac{1}{2.02\times 10^5}+\frac{1}{2.02\times 10^5}\right] \geq 1.212$$

$$\therefore \quad \sigma_{es}^2\ \geq\ 501072.24$$

$$\therefore \quad \sigma_{es} \geq 707.9\ MPa$$

$$i.e., (2.75\ \times BHN\ - 70) \geq 707.9$$

$$i.e., BHN \geq 282.86$$

OR

From the Table (T 12.16)(P-193) for $\alpha = 20^0$

$$k\ \geq 1.212\ MPa$$

Surface hardness no for pinion = 350 BHN

Surface hardness no for gear = 250 BHN

Hence suggested average surface hardness for the gear pair $\geq 282.86\ \ BHN$

Hence if the average surface hardness of the gear pair is more than 282.86 BHN, then the design will be satisfactory from the stand point of wear or durability also.

3. A pair of straight teeth spur gears, having 20^0 involutes full depth teeth is to transmit 12 kW at 300 rpm of the pinion. The speed ration is 3 : 1. The allowable static stress for a gear of CI and pinion of steel are 60 MPa and 105 MPa respectively. Assume number of teeth of pinion = 16 ; face width = 14 times the module ; velocity $(Cv) = \frac{4.58}{4.58+v}$, v being the pitch line velocity in m/s and tooth form factor $(y) = 0.154 - \frac{0.912}{No.of\ teeth}$. design the spur gear and check the gear for wear given $\sigma_{es} = 600\ MPa, E_P = 202\ kN/mm^2, E_G = 100\ kN/mm^2$.
(Dec 2011)(Marks 15)

Data: $\propto = 20^0\ FDI\ ; P = 12\ kW\ ;\ n_1 = 300\ rpm\ ;\ i = {}^3/_1\ ;$

$$\sigma_{d_1} = 105\ MPa;\ \sigma_{d_2} = 60\ MPa\ ; z_1 = 16\ teeth\ ;\ E_1 = 202\ GPa;$$

$$E_2 = 100\ GPa\ ; b = 14m; \sigma_{es} = 600\ MPa;$$

$$(C_v) = \frac{4.58}{4.58+v}\ ;\ (y) = 0.154 - \frac{0.912}{No.of\ Teeth}.$$

Solution: *Note: - Since the diameters are unknown, it is possible to design for smallest pitch diameter.*

Velocity ratio (i)

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_1}{d_2}$$

$$3 = \frac{300}{n_2}$$

$$\therefore \quad n_2 = 100$$

$$\therefore\ z_2 = i\ z_1 = 3 \times 16 = 48\ Teeth$$

To determine 'y' the lewis form factor

For 20^0 FDI profile

$$y = 0.154 - \frac{0.912}{Z} \quad for\ 20^0\ Involute\ system.$$

$$(E - 12.17b)(P - 163) \quad (General\ equation)$$

$$For\ Pinion, \qquad y_1 = 0.154 - \frac{0.912}{Z_1}$$

$$y_1 = 0.154 - \frac{0.912}{16}$$

$$y_1 = 0.097$$

$$For\ Gear, \qquad y_2 = 0.154 - \frac{0.912}{Z_2}$$

$$y_2 = 0.154 - \frac{0.912}{48}$$

$$y_2 = 0.135$$

Identifying the weaker member

Particulars	σ_d	y	$\sigma_d\ y$	Remarks
Pinion	105	0.097	10.185	
gear	60	0.135	8.1	**Weaker**

Since $\sigma_{d_1}\ y_1 < \sigma_{d_2}\ y_2$, $Pinion\ is\ weaker\ member.$

Design should be based on Pinion (weaker member).

Tangential tooth load (F_t)

$F_t = \frac{1000 \times P \times C_s}{v}$ ... ... $(E\ 12.20\ a)(P - 164\ )$ $(General\ equation)$

$$F_{t_2} = \frac{1000\ \times P\ \times C_{s_2}}{v_2} \qquad (Design\ equation)$$

Where, P = 12 kW= Power in kW.

$C_s = Service\ factor\ form\ \ From\ (T\ 12.8)(P - 187)$

$Assume\ medium\ shock\ and\ 8 - 10\ hours\ duty\ per\ day$

$C_s = 1.5$; $Speed = n_1 = rpm$

$v = Pitch\ line\ velocity\ of\ weaker\ member,$ m/s

$$v = \frac{\pi\ d\ n}{60000} = \frac{\pi\ (m\ Z)\ n}{60000}, \quad m/s \quad (\therefore d = m\ Z\ ) \qquad (General\ equation)$$

$$v_2 = \frac{\pi\ d_2\ n_2}{60000} = \frac{\pi\ (m\ Z_2)\ n_2}{60000} = \frac{\pi\ \times (m\ \times 48\ )\ \times 100}{60000}$$

$$v_2 = 0.251\ m \qquad m/s$$

$$\therefore \quad F_{t_2} = \frac{1000\ \times 12\ \times 1.5}{0.251\ m}$$

$$F_{t_2} = \frac{71713.15}{m} \quad N \qquad \dots \dots \dots Equn\ (i)$$

(ii) Tangential tooth load from lewis equation:

$F_t = \sigma_d\ C_v\ b\ y\ p$ ... ... $(E\ 12.15)(P\ 163\ )$ $(General\ equation)$

$$F_{t_2} = \sigma_{d_2}\ C_{v_2}\ b\ y_2\ p \qquad (Design\ equation)$$

Where, $\sigma_{d_2} =\ allowable\ static\ stress\ of\ the\ pinion\ = 60\ MPa$

Given face width, b = 14 m

y_1 = Lewis form factor of the Pinion

$y_2 = 0.135$

$Circular\ Pitch, \qquad p = \pi\ m$

$$F_{t_2} = 60 \times\ C_{v_2}\ \times\ (14\ \times m)\ \times 0.135\ \times (\pi\ \times m)$$

$$F_{t_2} = 356.26\ m^2\ C_{v_2} \quad N \qquad \dots \dots \dots\ Equn\ (ii)$$

Equating equation (*i*) & (*ii*)

$$356.26\ m^2\ C_{v_2} = \frac{71713.15}{m}$$

$$m^3\ C_{v_2} = 201.3 \qquad \dots \dots \dots\ Equn\ (iii)$$

To find module 'm'

Trial 1:- *Let select 'm =5'*

$Velocity \quad v_2 = 0.251\ m = 0.251 \times 5$

$v_2 = 1.255 \quad m/s$

$Velocity\ factor\ [\ Barth's formula\ ]\ \ (C_v)$

$C_v = \frac{4.58}{4.58 + v}$ $for\ v \leq 8\ m/s$ ... ... $(E\ 12.19\ a)(P\ 164\ )$ $(General\ equation)$

$$C_{v_2} = \frac{4.58}{4.58 + v_2} \qquad (Design\ equation)$$

$$C_{v_2} = \frac{4.58}{4.58 + 1.255}$$

$$C_{v_2} = 0.7849$$

Now from equation ..... (*iii)*

$$m^3\, C_{v_2} \geq 201.3 \qquad \dots\dots\dots\ Equn\ (iii)$$

$$5^3 \times 0.7849 = 201.3$$

$$88.5625 < 201.3 \qquad \therefore not\ suitable.$$

Trial 2:- *Let select 'm =6'*

$$Velocity\ \ v_2 = 0.251\, m = 0.251 \times 6$$

$$v_2 = 1.506 \quad {}^{m}/_{s}$$

$$Velocity\ factor\ [\ Barth's formula\ ]\ \ (C_v)$$

$$C_{v_2} = \frac{4.58}{4.58 + v_2} \qquad (Design\ equation)$$

$$C_{v_2} = \frac{4.58}{4.58 + 1.506}$$

$$C_{v_2} = 0.67$$

Now from equation ..... (*iii)*

$$m^3\, C_{v_2} \geq 201.3 \qquad \dots\dots\dots\ Equn\ (iii)$$

$$6^3 \times 0.67 = 201.3$$

$$144.72 < 201.3 \qquad \therefore not\ suitable.$$

$$\therefore Take \qquad m = \ mm$$

Trial 3:- *Let select 'm =7'*

$$Velocity\ \ v_2 = 0.251\, m = 0.251 \times 7$$

$$v_2 = 1.757 \quad {}^{m}/_{s}$$

$$Velocity\ factor\ [\ Barth's formula\ ]\ \ (C_v)$$

$$C_{v_2} = \frac{4.58}{4.58 + v_2} \qquad (Design\ equation)$$

$$C_{v_2} = \frac{4.58}{4.58 + 1.757}$$

$$C_{v_2} = 0.6345$$

Now from equation ..... (*iii)*

$$m^3\, C_{v_2} \geq 201.3 \qquad \dots\dots\dots\ Equn\ (iii)$$

$$7^3 \times 0.6345 = 201.3$$

$$217.63 > 201.3 \qquad \therefore suitable.$$

$$\therefore Take \qquad m = 7\ mm$$

∴ Face width 'b' b = 14 m = 14 x7 = 98 mm.

$$\therefore \quad F_{t_2} = \frac{71713.15}{m} = \frac{71713.15}{7} = 10244.74\ N$$

Check for the stress

Allowable stress by the equation

$$\sigma_{allow} = (\sigma_d\, C_v)_{allow} \qquad (General\ equation)$$

$$\sigma_{allow} = \left(\sigma_{d_2}\, C_{v_2}\right)_{allow} \qquad (Design\ equation)$$

$$\sigma_{allow} = 60 \times 0.6345$$

$$\sigma_{allow} = 38.07\ MPa$$

Induced stress

$$\sigma_{ind} = (\sigma_d\, C_v)_{ind} = \frac{F_t}{b\, y\, p} \quad \ldots\ldots (E\ 12.15)(P\ 163\ ) \quad (General\ equation)$$

$$\sigma_{ind} = (\sigma_d\, C_v)_{ind} = \frac{F_{t_2}}{b\, y_2\, p} \qquad (Design\ equation)$$

$$\sigma_{ind} = \frac{10244.74}{98 \times 0.135\ \times \pi \times 7}$$

$$\sigma_{ind} = 35.212\ MPa$$

If $\sigma_{ind} < \sigma_{allow}$ ($35.212MPa < 38.07\ MPa$) , then the design is satisfactory.

Dimensions

Calculate all important geometric parameter of tooth profile by using the equations given in Table (T 12.4)(P 183)

Properties of involutes teeth

Tooth Characteristics		20^0 FDI	*Pinion*	*Gear*
Addendum	h_a	m	7	7
Dedendum	h_f	1.57 m	10.99	10.99
Thickness of the tooth	t	1.571 m	10.997	10.997
No of Teeth	z		16	48
Pitch circle diameter	d	m z	48	144

Checking for dynamic load

Acc to bucking ham's equation

Dynamic load(F_d)

$$F_d = F_t + \frac{K_3\, v\, (C\, b\ +\ F_t)}{K_3\, v\ + \sqrt{(C\, b\ +\ F_t)}} \quad \ldots\ldots (E\ 12.33)(P\ 166\ ) \qquad (General\ equation)$$

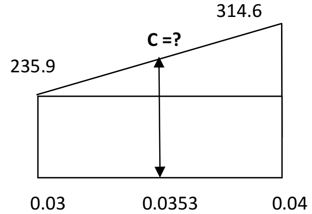
$$F_d = F_{t_2} + \frac{K_3\, v_2\, \left(C\, b\ +\ F_{t_2}\right)}{K_3\, v_2\ + \sqrt{\left(C\, b\ +\ F_{t_2}\right)}} \qquad (Design\ equation)$$

From Table T(12.13) (P- 191) for carefully cut gears for m = 7 mm

Machining error, e= 0.0353mm

From T(12.12) (P- 190) for e =0.0353 mm $\alpha = 20^0\ FDI$ and steel pinion – C I gear

Error 'e'	C
0.03	235.9
0.0353	?
0.04	314.6



0.03	235.9
0.0353	C

$$C\ \times 0.03 = \ 235.9 \times 0.0353$$

$$C = \frac{235.9\ \times 0.0353}{0.03}$$

$$C = 277.57\ MPa$$

Where, $F_{t_1} = 10244.74\ N$; $K_3 = 20.67$; $v_2 = 1.757\ {}^{m}/_{s}$;

$$C = 277.57 \text{ MPa};\ b = 98 \text{ mm};$$

$$F_d = 10244.74 + \frac{20.67 \times 1.757 \times (277.57 \times 98 + 10244.74)}{20.67 \times 1.757 + \sqrt{(277.57 \times 98 + 10244.74)}}$$

$$F_d = 10244.74 + \frac{1359955.3}{229.83}$$

$$F_d = 10244.74 + 5917.22$$

$$F_d = 16162\ N$$

Check for endurance strength(F_{en})

$$F_{en} = \sigma_{en} b\, Y\, m \quad \ldots\ldots (E\ 12.34)(P\ 166) \quad (General\ equation)$$

$$F_{en_1} = \sigma_{en_2} b\, Y_2\, m \quad (Design\ \ equation)$$

In lewis equation the allowable bending stress is approximately $\frac{1}{3}$ of ultimate stress.

∴ Ultimate stress of the pinion material(σ_u),

$$\sigma_u = 3 \times \sigma_{d_2} = 3 \times 60 = 180\ MPa$$

Now endurance limit $\sigma_{en} = 0.5 \times \sigma_u = 0.5 \times 180 = 90\ MPa$

Where, $\sigma_{en} = 90\ MPa$; $b = 98\ mm$; $Y_2 = \pi\, y_2 = \pi \times 0.135 = 0.424\ {}^{m}/_{s}$; m = 7 mm.

$$F_{en_2} = 90 \times 98 \times 0.424 \times 7$$

$$F_{en_2} = 26177.76\ N$$

Since $F_{en} > F_d$ ($26177.76\ N > 16162\ N$), the design will be ***satisfactory from the point of wear or durability.***

Check for wear load (F_w)

Acc to bucking ham's equation wear load

$$F_w = d_1\, b\, Q\ k \quad \ldots\ldots (E\ 12.36\ a)(P\ 167) \quad (General\ equation)$$

$$F_w = d_1\, b\, Q\ k \quad (Design\ equation)$$

Where ratio factor

$$Q = \frac{2\, d_2}{d_2 + d_1} = \frac{2\, z_2}{z_2 + z_1} \quad \ldots\ldots (E\ 12.36\ c)(P\ 167)$$

$$Q = \frac{2\, z_2}{z_2 + z_1} = \frac{2 \times 48}{48 + 16}$$

$$Q = 1.5$$

Surface endurance limit

$$\sigma_{es} = (2.75\, BHN - 70) \quad MPa \quad \ldots\ldots (E\ 12.36\ d)(P\ 167)$$

$$\sigma_{es} = 600\ MPa$$

$$600 = (2.75\, BHN - 70)$$

$$BHN = 243.64$$

Load stress factor $k = \frac{\sigma_{es}^2\, Sin \propto}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$ ……(E 12.36 b)(P 167)

$$k = \frac{600^2\, Sin\ 20}{1.4}\left[\frac{1}{1 \times 10^5} + \frac{1}{2.02 \times 10^5}\right]$$

$$k = 1.315 \quad MPa$$

$$d_1 = m\, Z_1 = m \times 16 = 7 \times 16$$

$$d_1 = 112\ mm$$

i.e., $F_w = d_1\, b\, Q\ k = 112 \times 98 \times 1.5 \times 1.315$

$$F_w = 21650.16\ N$$

Since $F_d < F_w$ $(16162\ N < 21650.16\ N)$, the design will be **satisfactory from the point of wear or durability.**

Hence suggested average surface hardness for the gear pair $\geq 250\ BHN$

Hence if the average surface hardness of the gear pair is more than 250 BHN, then the design will be satisfactory from the stand point of durability or wear also.

From the Table (T 12.16)(P- 193) for $\alpha = 20^0$

$$k \geq 1.315 MPa$$

Surface hardness no for Steel - Pinion = 250 BHN

Surface hardness no for C I - Gear = 180 BHN

4. A air compressor is driven by a 20 kW, 1200 rpm motor, trough a pair of $14\frac{1}{2}^0$ involute spur gears. The speed of the compressor is 300 rpm and the centre distance is approximately 400 rpm. The pinion is to be of cast steel, heat treated and the gear is of CI, grade 35. Assume the gears are subjected to medium shock and working 8 – 10 hrs/ day. Design the gears and check the design for wear load.

(Dec. 2010)(Marks 14)

Data:

$$\propto = 14\frac{1}{2}^0\ FDI\ ; P = 20\ kW\ ;\ d_1 = 105\ mm\ ;\ n_1 = 1200\ rpm\ ;\ n_2 = 300\ rpm\ ; c = 400\ mm;$$

$$Pinion\ material - Cast\ Steel\ ;$$

$$Gear\ material - Cast\ iron\ 35\ ;$$

allowable stress for

$$Pinion\ material - Cast\ Steel\ untreated\ ;\quad \sigma_{d_1} = 138.3\ MPa$$

$$Gear\ material - Cast\ iron\ ;\quad \sigma_{d_2} = 56.4\ MPa$$

Solution: *Velocity ratio (i)*

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_1}{d_2}$$

$$i = \frac{n_1}{n_2}$$

$$i = \frac{1200}{300}$$

$$\therefore\ Velocity\ ratio \qquad i = 4$$

$$Centre\ distance,\quad c = \frac{d_1 + d_2}{2} = \frac{d_1 + i\, d_1}{2} = \frac{d_1(1+i)}{2}$$

$$400 = \frac{d_1(1+4)}{2} = \frac{5 \times d_1}{2} \qquad \therefore (d_2 = i\, d_1)$$

$$d_1 = 160\ mm$$

$$\therefore\ Pitch\ circle\ diameter\ of\ Pinion \qquad d_1 = 160\ mm$$

$$\therefore d_2 = i\, d_1\ ; \quad d_2 = 4 \times 160$$

$$\therefore\ Pitch\ circle\ diameter\ of\ Gear \qquad d_2 = 640\ mm$$

To determine 'y' the lewis form factor

$$y = 0.124 - \frac{0.684}{Z} \quad for\ 14\frac{1}{2}^{0}\ FDI\ system.$$

$$(E - 12.17a)(\ P - 163) \quad (General\ equation)$$

Temporarily Assume $Z_1 = 20\ Teeths$

$$Z_1 = 20\ Teeths, \qquad \therefore Z_2 = 4 \times Z_1 = 4 \times 20 = 80\ Teeths$$

$$For\ Pinion\ y_1 = 0.154 - \frac{0.912}{Z_1}$$

$$y_1 = 0.124 - \frac{0.684}{20}$$

$$y_1 = 0.0898$$

$$For\ Gear \quad y_2 = 0.124 - \frac{0.684}{Z_2}$$

$$y_2 = 0.124 - \frac{0.684}{80}$$

$$y_2 = 0.11545$$

Identifying the weaker member

Particulars	σ_d	y	$\sigma_d\ y$	Remarks
Pinion	138.3	0.0898	12.42	
Gear	56.4	0.11545	6.51138	**Weaker**

Since $\sigma_{d_2}\, y_2 < \sigma_{d_1}\, y_1$, $Gear\ is\ weaker\ member.$

Design should be based on Gear (weaker member).

Tangential tooth load

$$F_t = \frac{1000 \times P \times C_s}{v} \quad \ldots\ldots (E\ 12.20\ a)(P - 164) \quad (General\ equation)$$

$$F_{t_2} = \frac{1000 \times P \times C_{s_2}}{v_2} \qquad (Design\ equation)$$

Where, P = 20 kW= Power in kW.

$$C_s = Service\ factor\ form\ From\ (T\ 12.8)(P - 187)$$

$Assume\ medium\ shock\ and\ 8 - 10\ hours\ duty\ per\ day$

$$C_{s_2} = 1.5\ ;\ Speed = n_2 = 300\ rpm$$

$$v = Pitch\ line\ velocity\ of\ weaker\ member, \qquad {}^{m}/_{s}$$

$$v = \frac{\pi\, d\, n}{60000} = \frac{\pi\,(m\,Z)\,n}{60000}, \quad {}^{m}/_{s}\ (\therefore d = m\,Z) \quad (General\ equation)$$

$$v_2 = \frac{\pi\, d_2\, n_2}{60000} = \frac{\pi \times 640 \times 300}{60000}$$

$$v_2 = 10.053 \qquad {}^{m}/_{s}$$

$$\therefore \quad F_{t_2} = \frac{1000 \times 20 \times 1.5}{10.053}$$

$$F_{t_2} = 2984.2\ \ N \qquad \ldots\ldots\ldots Equn\ (i)$$

(ii) Tangential tooth load from lewis equation:

$$F_t = \sigma_d \, C_v \, b \, y \, p \quad \ldots\ldots(E\ 12.15)(P\ 163) \quad (General\ equation)$$

$$F_{t_2} = \sigma_{d_2} \, C_{v_2} \, b \, y_2 \, p \quad (Design\ equation)$$

Where $\sigma_{d_2} = \ allowable\ static\ stress\ of\ the\ pinion \ = 55\ MPa$

$$\text{b} = \text{face width} \quad 9.5\text{m} \le \text{b} \le 12.5\text{ m} \quad \ldots\ldots(E\ 12.18)(P\ 164)$$

$$\text{Take face width,} \quad \text{b} = 10\text{ m}$$

$$Circular\ Pitch, \quad p = \pi m$$

$$Velocity \quad v_2 = 10.053 \ ^m/_s$$

$Velocity\ factor\ [\ Barth's formula\ ]\ (\ C_v)$

$$C_v = \frac{4.58}{4.58 + v} \quad for\ v \le 13\ ^m/_s$$

$$\ldots\ldots(E\ 12.19\ a)(P\ 164)\ (General\ equation)$$

$$C_{v_2} = \frac{4.58}{4.58 + v_2} \quad (Design\ equation)$$

$$C_{v_2} = \frac{4.58}{4.58 + 10.053}$$

$$C_{v_2} = 0.313$$

y_2 = Lewis form factor of the Pinion

$$For\ Gear \quad y_2 = 0.124 - \frac{0.684}{Z_2}$$

$$y_2 = 0.124 - \frac{0.684}{\left(\frac{d_2}{m}\right)}$$

$$y_2 = 0.124 - \frac{0.684 \times m}{d_2}$$

$$y_2 = 0.124 - \frac{0.684 \times m}{640}$$

$$y_2 = 0.124 - 1.06875 \times 10^{-3} m$$

$$F_{t_2} = 56.4 \times 0.313 \times (10 \times m) \times y_2 \times (\pi \times m)$$

$$F_{t_2} = 554.6\, m^2 y_2 \quad N \quad \ldots\ldots\ldots\ Equn\ (ii)$$

Equating equation (*i*) & (*ii*)

$$554.6\, m^2 y_2 = 2984.2$$

$$m^2 y_2 \ge 5.38 \quad \ldots\ldots\ldots\ Equn\ (iii)$$

To find module 'm'

Trial 1:- Let select ' m = 5'

$$y_2 = 0.124 - 1.06875 \times 10^{-3} m$$

$$y_2 = 0.124 - 1.06875 \times 10^{-3} \times 5$$

$$y_2 = 0.11866$$

Now from equation ..... (*iii)*

$$m^2 y_2 \ge 9.1956 \quad \ldots\ldots\ldots\ Equn\ (iii)$$

$$5^2 \times 0.11866 \ge 5.38$$

$$2.9665 < 5.38 \quad \therefore not\ suitable.$$

Trial 2:- *Let select ' m = 6'*

$$y_2 = 0.124 - 1.06875 \times 10^{-3} m$$

$$y_2 = 0.124 - 1.06875 \times 10^{-3} \times 6$$

$$y_2 = 0.11759$$

Now from equation ..... (*iii)*

$$m^2 y_2 \geq 9.1956 \qquad \ldots\ldots\ldots\ Equn\ (iii)$$

$$6^2 \times 0.11759 \geq 5.38$$

$$4.233 < 5.38 \qquad \therefore not\ suitable.$$

Trial 1:- *Let select ' m = 7'*

$$y_2 = 0.124 - 1.06875 \times 10^{-3} m$$

$$y_2 = 0.124 - 1.06875 \times 10^{-3} \times 7$$

$$y_2 = 0.11652$$

Now from equation ..... (*iii)*

$$m^2 y_2 \geq 9.1956 \qquad \ldots\ldots\ldots\ Equn\ (iii)$$

$$7^2 \times 0.11652 \geq 5.38$$

$$5.7095 > 5.38 \qquad \therefore suitable.$$

$$\therefore Take \qquad m = 7\ mm$$

$y_2 = 0.11652;$ $\quad y_2 = 0.124 - \frac{0.684}{Z_2}$

$$\therefore 0.11652 = 0.124 - \frac{0.684}{Z_2}$$

∴ Face width 'b' b = 10 m = 10 x 7 = 70 mm.

Dimensions

Calculate all important geometric parameter of tooth profile by using the equations given in Table (T 12.4)(P 183)

Properties of involutes teeth

Tooth Characteristics		20^0 FDI	$Pinion$	$Gear$
Addendum	h_a	m	7	7
Dedendum	h_f	1.57 m	11	11
Thickness of the tooth	t	2.571 m	18	18
No of Teeth	z		24	92
Pitch circle diameter	d	m z	160	640

Check for the stress

Allowable stress by the equation

$$\sigma_{allow} = (\sigma_d\, C_v)_{allow} \qquad (General\ equation)$$

$$\sigma_{allow} = \left(\sigma_{d_2}\, C_{v_2}\right)_{allow} \qquad (Design\ equation)$$

$$\sigma_{allow} = 56.4 \times 0.313$$

$$\sigma_{allow} = 17.65\ MPa$$

induced stress

$$\sigma_{ind} = (\sigma_d \, C_v)_{ind} = \frac{F_t}{b \, y \, p}$$

$$\ldots\ldots(E\ 12.15)(P\ 163\ ) \qquad (General\ equation)$$

$$\sigma_{ind} = (\sigma_d \, C_v)_{ind} = \frac{F_{t_2}}{b \, y_2 \, p} \qquad (Design\ equation)$$

$$\sigma_{ind} = \frac{2984.2}{70 \times 0.11652 \times \pi \times 7}$$

$$\sigma_{ind} = 15.73\ MPa$$

If $\sigma_{ind} < \sigma_{allow}(15.73\ MPa < 17.65\ MPa)$, then the design is satisfactory.

Checking for dynamic load

Acc to bucking ham's equation

Dynamic load

$$F_d = F_t + \frac{K_3 \, v \, (C\, b + F_t)}{K_3 \, v + \sqrt{(C\, b + F_t)}} \quad \ldots\ldots(E\ 12.33)(P\ 166\ )\ (General\ equation)$$

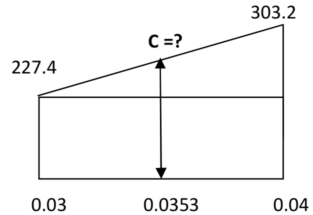
$$F_d = F_{t_2} + \frac{K_3 \, v_2 \left(C\, b + F_{t_2}\right)}{K_3 \, v_2 + \sqrt{\left(C\, b + F_{t_2}\right)}} \qquad (Design\ equation)$$

From Table T(12.13) (P- 191) for carefully cut gears for m = 7 mm

Machining error, e= 0.0353 mm

From T(12.12) (P- 190) for e = 0.0353 mm $\alpha = 14\frac{1}{2}^{0}\ FDI$ and steel pinion – C I - gear

Error 'e'	C
0.03	227.4
0.0353	?



0.03	227.4
0.0353	C

$$C \times 0.03 = 227.4 \times 0.0353$$

$$C = \frac{227.4 \times 0.0353}{0.03}$$

$$C = 267.574\ MPa$$

Where, $F_{t_2} = 2984.2\ N$; $K_3 = 20.67$; $v_2 = 10.053\ \ m/_{S}$;

$$\mathrm{C} = 267.574\ \mathrm{MPa};\ \mathrm{b} = 70\ \mathrm{mm};$$

$$F_d = 2984.2 + \frac{20.67 \times 10.053 \times (267.574 \times 70 + 2984.2\,)}{20.67 \times 10.053 + \sqrt{(267.574 \times 70 + 2984.2\,)}}$$

$$F_d = 2984.2 + \frac{4512150.67}{355.15}$$

$$F_d = 2984.2 + 12705$$

$$F_d = 15689.2\ N$$

Check for endurance strength(F_{en})

$$F_{en} = \sigma_{en} b\, Y\, m \qquad \ldots\ldots(E\ 12.34)(P\ 166\ ) \qquad (General\ equation)$$

$$F_{en_2} = \sigma_{en_2} b\, Y_2\, m \qquad (Design\ \ equation)$$

In lewis equation the allowable bending stress is approximately $\frac{1}{3}$ of ultimate stress.

∴ ultimate stress of the pinion material, $\sigma_u = 3 \times \sigma_{d_2} = 3 \times 56.4 = 169.2\ MPa$

Now endurance limit $\sigma_{en} = 0.5 \times \sigma_u = 0.5 \times 169.2 = 84.6\ MPa$

Where, $\sigma_{en} = 82.5\ MPa$; $b = 100\ mm$; $Y_2 = \pi\ y_2 = \pi \times 0.123252 = 0.3872\ {}^{m}/_{s}$; m = 7 mm.

$$F_{en_1} = 84.6 \times 70 \times 0.3872 \times 7$$

$$F_{en_1} = 16051\ N$$

Since $F_{en} > F_d$ $(16051\ N > 15689.2\ N)$, the design will be ***satisfactory from the point of wear or durability.***

Check for wear load

Acc to bucking ham's equation wear load

$$F_w = d_1\ b\ Q\ k \quad \ldots\ldots (E\ 12.36\ a)(P\ 167) \quad (General\ equation)$$

$$F_w = d_1\ b\ Q\ k \quad (Design\ equation)$$

Where ratio factor

$$Q = \frac{2\ d_2}{d_2 + d_1} = \frac{2\ z_2}{z_2 + z_1} \quad \ldots\ldots (E\ 12.36\ c)(P\ 167)$$

$$Q = \frac{2\ d_2}{d_2 + d_1} = \frac{2 \times 640}{640 + 160}$$

$$Q = 1.6$$

From table (T-12.15)(P-192) for $\sigma_{en} = 84.6\ MPa$ Core (BHN) = 160 BHN $(Brinell\ hardness\ Number)$

From table (T-1.1)(P-7) for carbon steel & C I

Young's modulus $E_1 = 202\ GPa = 202 \times 10^3\ MPa$

$$E_2 = 100\ GPa = 100 \times 10^3\ MPa$$

Surface endurance limit

$$\sigma_{es} = (2.75\ BHN - 70)\ MPa \quad \ldots\ldots (E\ 12.36\ d)(P\ 167)$$

$$\sigma_{es} = (2.75 \times 160 - 70)$$

$$\sigma_{es} = 370\ MPa$$

Load stress factor

$$k = \frac{{\sigma_{es}}^2\ Sin \propto}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right] \quad \ldots\ldots (E\ 12.36\ b)(P\ 167)$$

$$k = \frac{370^2\ Sin\ 14.5}{1.4}\left[\frac{1}{2.02 \times 10^5} + \frac{1}{1.00 \times 10^5}\right]$$

$$k = 0.366\ MPa$$

i.e., $F_w = d_1\ b\ Q\ k = 160 \times 70 \times 1.6 \times 0.366$

$$F_w = 6558.72\ N$$

Since $F_w < F_d$ $(6558.72\ N < 15689.2\ N)$, the design will be **unsatisfactory from the point of wear or durability.**

For safer design

$$F_w \geq F_d$$

$$i.e., d_1\, b\, Q\;\; k \;\geq\; F_d$$

$$160 \times 70 \times 1.6 \times k \;\geq\; 15689.2$$

$$\therefore Load\ stress\ factor\;\; k \geq 0.8755$$

$$i.e., \frac{\sigma_{es}{}^2\, Sin\, \propto}{1.4}\left[\frac{1}{E_1}+\frac{1}{E_2}\right] \geq 0.8755$$

$$i.e., \frac{\sigma_{es}{}^2\, Sin\, 14.5}{1.4}\left[\frac{1}{2.02\times10^5}+\frac{1}{1.00\times10^5}\right] \geq 0.8755$$

$$\therefore\;\; \sigma_{es}{}^2 \;\geq\; 327438$$

$$\therefore\;\; \sigma_{es} \geq 572\ MPa$$

$$i.e., (2.75\ \times BHN\ - 70) \geq 572$$

$$i.e., BHN \geq 233$$

OR

From the Table (T 12.16)(P-193) for $\alpha = 14.5^0$

$$k\ \geq 0.8755\ MPa$$

Surface hardness no for Steel - Pinion = 250 BHN

Surface hardness no for C I - Gear = 180 BHN

Hence suggested average surface hardness for the gear pair $\geq 250\;\; BHN$

Hence if the average surface hardness of the gear pair is more than 250 BHN, then the design will be satisfactory from the stand point of wear or durability also.

5. A spur pinion of cast steel $(\sigma_d = 140\;\; MN/m^2)$ is to drive a spur gear of cast Iron $(\sigma_d = 55\;\; MN/m^2)$. The transmission ratio is to be $2\frac{1}{2}$ to 1. The diameter of the pinion is to be 105 mm & the teeth are full depth involutes form. Design for the greatest number of teeth. Determine the necessary module and face width of the gears for strength only. Power to be transmitted is 20 kW at 900 rpm of the pinion.

(Dec.08/Jan. 09)(Marks 15)

Data: assume tooth profile system = $20^0\ FDI$

$$\propto = 20^0\ FDI\ ; P = 20\ kW\ ;\ d_1 = 105\ mm\ ;\ n_1 = 900\ rpm\ ;$$

$$Pinion\ material - Cast\ Steel\ ;$$

$$Gear\ material - Cast\ iron\ ;$$

allowable stress for

$$Pinion\ material - Cast\ Steel\ ;\quad \sigma_{d_1} = 140\ MPa$$

$$Gear\ material - Cast\ iron\ ;\quad \sigma_{d_2} = 55\ MPa$$

Solution: *Note:* - the power to be transmitted the rpm of the pinion and pitch diameter of pinion are also specified and therefore the tangential load to be transmitted at the pitch point from pinion to gear may be strained. Equating this to beam strength of tooth, the required module may be calculated. While selecting a standard module, check that the number of teeth obtained for these values of module for both pinion and gear. Should be whole no's using the information of the transmission ratio (ang velocity ratio)

Velocity ratio (i)

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_1}{d_2}$$

$$2.5 = \frac{d_1}{d_2}$$

$$2.5 = \frac{105}{d_2}$$

$$\therefore \; Diameter \; of \; Gear \qquad d_2 = 262.5 \; \approx 263 \; mm$$

$$i = \frac{n_1}{n_2}$$

$$2.5 = \frac{900}{n_2}$$

$$2.5 = \frac{105}{d_2}$$

$$\therefore \; Speed \; of \; Gear \qquad n_2 = 360 \; rpm$$

To determine 'y' the lewis form factor

For 20⁰ FDI profile

$$y = 0.154 - \frac{0.912}{Z} \quad for \; 20^0 \; FDI \; system.$$

$$(E - 12.17b)(\; P - 163) \quad (General \; equation)$$

Temporarily Assume $Z_1 = 20 \; Teeths$

$$Z_1 = 20 \; Teeths, \qquad \therefore Z_2 = 2.5 \; \times \; Z_1 = 2.5 \; \times 20 = 50 \; Teeths$$

$$For \; Pinion \; y_1 = 0.154 - \frac{0.912}{Z_1}$$

$$y_1 = 0.154 - \frac{0.912}{20}$$

$$y_1 = 0.1084$$

$$For \; Gear \qquad y_2 = 0.154 - \frac{0.912}{Z_2}$$

$$y_2 = 0.154 - \frac{0.912}{80}$$

$$y_2 = 0.13576$$

Identifying the weaker member

Particulars	σ_d	y	$\sigma_d \; y$	Remarks
Pinion	140	0.1084	15.176	
Gear	55	0.13576	7.4668	**Weaker**

Since $\sigma_{d_2} \, y_2 \; < \sigma_{d_1} \, y_1$, $\quad Gear \; is \; weaker \; member.$

Design should be based on Gear (weaker member).

Tangential tooth load

$$F_t = \frac{1000 \; \times P \; \times C_s}{v} \quad \ldots \ldots (E \; 12.20 \; a)(P - 164 \;) \qquad (General \; equation)$$

$$F_{t_2} = \frac{1000 \; \times P \; \times C_{s_2}}{v_2} \qquad (Design \; equation)$$

Where, P = 20 kW= Power in kW.

$$C_s = Service \; factor \; form \; \; From \; (T \; 12.8)(P - 187)$$

$Assume \; medium \; shock \; and \; 8 - 10 \; hours \; duty \; per \; day$

$$C_{s_2} = 1.5 \quad ; \; Speed = \; n_2 = 360 \; rpm$$

$$v = Pitch \; line \; velocity \; of \; weaker \; member, \qquad {}^{m}/_{s}$$

$$v = \frac{\pi \; d \; n}{60000} = \frac{\pi \; (m \; Z) \; n}{60000} \; , \qquad {}^{m}/_{s} \; \; (\therefore d = m \; Z \;) \qquad (General \; equation)$$

$$v_2 = \frac{\pi\, d_2\, n_2}{60000} = \frac{\pi \times 263 \times 360}{60000}$$

$$v_2 = 4.957 \quad {}^{m}/_{s}$$

$$\therefore \quad F_{t_2} = \frac{1000 \times 20 \times 1.5}{4.957}$$

$$F_{t_2} = 6052.05 \quad N \qquad \ldots\ldots\ldots Equn\ (i)$$

(ii) Tangential tooth load from lewis equation:

$$F_t = \sigma_d\, C_v\, b\, y\, p \qquad \ldots\ldots (E\ 12.15)(P\ 163) \qquad (General\ equation)$$

$$F_{t_2} = \sigma_{d_2}\, C_{v_2}\, b\, y_2\, p \qquad (Design\ equation)$$

Where $\sigma_{d_2} = \ allowable\ static\ stress\ of\ the\ pinion \ = 55\ MPa$

b = face width 9.5m ≤ b ≤ 12.5 m $\ldots\ldots (E\ 12.18)(P\ 164)$

Take face width, b = 10 m

$Circular\ Pitch, \qquad p = \pi\, m$

$Velocity \quad v_2 = 4.957 \ {}^{m}/_{s}$

$Velocity\ factor\ [\ Barth's formula\ ]\ (\ C_v)$

$$C_v = \frac{3.05}{3.05 + v} \qquad for\ v \le 8\ {}^{m}/_{s}$$

$$\ldots\ldots (E\ 12.19\ a)(P\ 164)\ (General\ equation)$$

$$C_{v_2} = \frac{3.05}{3.05 + v_2} \qquad (Design\ equation)$$

$$C_{v_2} = \frac{3.05}{3.05 + 4.957}$$

$$C_{v_2} = 0.3809$$

y_2 = Lewis form factor of the Pinion

$$For\ Gear \qquad y_2 = 0.154 - \frac{0.912}{Z_2}$$

$$y_2 = 0.154 - \frac{0.912}{\left(\frac{d_2}{m}\right)}$$

$$y_2 = 0.154 - \frac{0.912 \times m}{d_2}$$

$$y_2 = 0.154 - \frac{0.912 \times m}{263}$$

$$y_2 = 0.154 - 3.4677 \times 10^{-3} m$$

$$F_{t_2} = 55 \times 0.3809 \times (10 \times m) \times y_2 \times (\pi \times m)$$

$$F_{t_2} = 658.148\, m^2 y_2 \quad N \qquad \ldots\ldots\ldots\ Equn\ (ii)$$

Equating equation (*i*) & (*ii*)

$$658.148\, m^2 y_2 = 6052.05$$

$$m^2 y_2 \ge 9.1956 \qquad \ldots\ldots\ldots\ Equn\ (iii)$$

To find module 'm'

Trial 1:- *Let select ' m = 5'*

$$y_2 = 0.154 - 3.4677 \times 10^{-3} m$$

$$y_2 = 0.154 - 3.4677 \times 10^{-3} \times 5$$

$$y_2 = 0.13666$$

Now from equation ….. (*iii*)

$$m^2 y_2 \ge 9.1956 \qquad \ldots\ldots\ldots\ Equn\ (iii)$$

$$5^2 \times 0.13666 \geq 9.1956$$

$$3.4165 < 9.1956 \qquad \therefore not\ suitable.$$

Trial 2:- *Let select ' m = 6'*

$$y_2 = 0.154 - 3.4677 \times 10^{-3} m$$

$$y_2 = 0.154 - 3.4677 \times 10^{-3} \times 6$$

$$y_2 = 0.1332$$

Now from equation ….. (*iii)*

$$m^2 y_2 \geq 9.1956 \qquad \ldots\ldots\ldots\ Equn\ (iii)$$

$$6^2 \times 0.1332 \geq 9.1956$$

$$4.795 < 9.1956 \qquad \therefore not\ suitable.$$

Trial 3:- *Let select ' m = 10'*

$$y_2 = 0.154 - 3.4677 \times 10^{-3} m$$

$$y_2 = 0.154 - 3.4677 \times 10^{-3} \times 10$$

$$y_2 = 0.119323$$

Now from equation ….. (*iii)*

$$m^2 y_2 \geq 9.1956 \qquad \ldots\ldots\ldots\ Equn\ (iii)$$

$$10^2 \times 0.119323 \geq 9.1956$$

$$11.9323 < 9.1956 \qquad \therefore suitable.$$

$y_2 = 0.119323;$ $\quad y_2 = 0.154 - \frac{0.912}{Z_2}$

$$\therefore 0.119323 = 0.154 - \frac{0.912}{Z_2}$$

$$Z_2 = 28\ teeth$$

$$\therefore Z_1 = \frac{Z_2}{2.5} = \frac{28}{2.5} = 12\ teeth$$

$$\therefore Take \qquad m = 10\ mm$$

∴ Face width 'b' b = 10 m = 10 x 10 = 100 mm.

Dimensions

Calculate all important geometric parameter of tooth profile by using the equations given in Table (T 12.4)(P 183)

Properties of involutes teeth

Tooth Characteristics		20^0 FDI	*Pinion*	*Gear*
Addendum	h_a	m	10	10
Dedendum	h_f	1.57 m	15.7	15.7
Thickness of the tooth	t	1.571 m	15.71	15.71
No of Teeth	z		12	28
Pitch circle diameter	d	m z	105	263

Check for the stress

Allowable stress by the equation

$$\sigma_{allow} = (\sigma_d\, C_v)_{allow} \qquad (General\ equation)$$

$$\sigma_{allow} = \left(\sigma_{d_2}\, C_{v_2}\right)_{allow} \qquad (Design\ equation)$$

$$\sigma_{allow} = 55 \times 0.3809$$

$$\sigma_{allow} = 21\ MPa$$

induced stress

$$\sigma_{ind} = (\sigma_d\, C_v)_{ind} = \frac{F_t}{b\, y\, p}$$

$$\dots\dots (E\ 12.15)(P\ 163) \qquad (General\ equation)$$

$$\sigma_{ind} = (\sigma_d\, C_v)_{ind} = \frac{F_{t_2}}{b\, y_2\, p} \qquad (Design\ equation)$$

$$\sigma_{ind} = \frac{6052.05}{100 \times 0.119323 \times \pi \times 10}$$

$$\sigma_{ind} = 16\ MPa$$

If $\sigma_{ind} < \sigma_{allow} (16\ MPa < 21\ MPa)$, then the design is satisfactory.

Checking for dynamic load

Acc to bucking ham's equation

Dynamic load

$$F_d = F_t + \frac{K_3\, v\,(C\, b + F_t)}{K_3\, v + \sqrt{(C\, b + F_t)}} \quad \dots\dots (E\ 12.33)(P\ 166) \ (General\ equation)$$

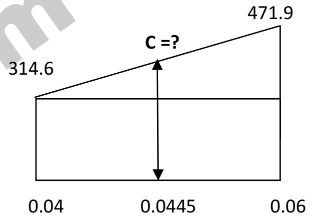
$$F_d = F_{t_2} + \frac{K_3\, v_2\,(C\, b + F_{t_2})}{K_3\, v_2 + \sqrt{(C\, b + F_{t_2})}} \qquad (Design\ equation)$$

From Table T(12.13) (P- 191) for carefully cut gears for m = 10 mm

Machining error, e= 0.0445 mm

From T(12.12) (P- 190) for e = 0.0455 mm $\alpha = 20^0\ FDI$ and steel pinion – C I - gear

Error 'e'	C
0.04	314.6
0.0445	?



0.04	314.6
0.0445	C

$$C \times 0.04 = 314.6 \times 0.0445$$

$$C = \frac{314.6 \times 0.0445}{0.04}$$

$$C = 350\ MPa$$

Where, $F_{t_2} = 6052.05\ N$; $K_3 = 20.67$; $v_2 = 4.957\ \ {}^{m}/_{s}$;

$$\mathrm{C} = 350\ \mathrm{MPa};\ \mathrm{b} = 100\ \mathrm{mm};$$

$$F_d = 6052.05 + \frac{20.67 \times 4.957 \times (350 \times 100 + 6052.05)}{20.67 \times 4.957 + \sqrt{(350 \times 100 + 6052.05)}}$$

$$F_d = 6052.05 + \frac{4206241.9}{305.07}$$

$$F_d = 6052.05 + 13787.8$$

$$F_d = 19839.85\ N$$

Check for endurance strength(F_{en})

$$F_{en} = \sigma_{en} b\, Y\, m \quad \ldots\ldots (E\ 12.34)(P\ 166) \quad (General\ equation)$$

$$F_{en_2} = \sigma_{en_2} b\, Y_2\, m \quad (Design\ \ equation)$$

In lewis equation the allowable bending stress is approximately $\frac{1}{3}$ of ultimate stress.

∴ ultimate stress of the pinion material, $\sigma_u = 3 \times \sigma_{d_2} = 3 \times 55 = 165\ MPa$

Now endurance limit $\sigma_{en} = 0.5 \times \sigma_u = 0.5 \times 165 = 82.5\ MPa$

Where, $\sigma_{en} = 82.5\ MPa$; $b = 100\ mm$; $Y_2 = \pi\, y_2 = \pi \times 0.119323 = 0.3746\ {}^{m}/_{s}$; m = 10 mm.

$$F_{en_1} = 82.5 \times 100 \times 0.3746 \times 10$$

$$F_{en_2} = 30904.5\ N$$

Since $F_{en} > F_d$ ($30904.5\ N > 19839.85\ N$), the design will be ***satisfactory from the point of wear or durability.***

Check for wear load

Acc to bucking ham's equation wear load

$$F_w = d_1\, b\, Q\, k \quad \ldots\ldots (E\ 12.36\ a)(P\ 167) \quad (General\ equation)$$

$$F_w = d_1\, b\, Q\, k \quad (Design\ equation)$$

Where ratio factor

$$Q = \frac{2\, d_2}{d_2 + d_1} = \frac{2\, z_2}{z_2 + z_1} \quad \ldots\ldots (E\ 12.36\ c)(P\ 167)$$

$$Q = \frac{2\, d_2}{d_2 + d_1} = \frac{2 \times 263}{263 + 105}$$

$$Q = 1.43$$

From table (T-12.15)(P-192) for $\sigma_{en} = 82.5\ MPa$ Core (BHN) = 160 BHN ($Brinell\ hardness\ Number$)

From table (T-1.1)(P-7) for carbon steel

Young's modulus $E_1 = 202\ GPa = 202 \times 10^3\ MPa$

$$E_2 = 100\ GPa = 100 \times 10^3\ MPa$$

Surface endurance limit

$$\sigma_{es} = (2.75\ BHN - 70) \quad MPa \quad \ldots\ldots (E\ 12.36\ d)(P\ 167)$$

$$\sigma_{es} = (2.75 \times 160 - 70)$$

$$\sigma_{es} = 370\ MPa$$

Load stress factor

$$k = \frac{\sigma_{es}^{\ 2}\ Sin\ \propto}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right] \quad \ldots\ldots (E\ 12.36\ b)(P\ 167)$$

$$k = \frac{370^2\ Sin\ 20}{1.4}\left[\frac{1}{2.02 \times 10^5} + \frac{1}{1.00 \times 10^5}\right]$$

$$k = 0.5 \quad MPa$$

i.e., $F_w = d_1\, b\, Q\, k = 105 \times 100 \times 1.43 \times 0.5$

$$F_w = 7507.5\ N$$

Since $F_w < F_d$ ($7507.5\ N < 19839.85\ N$), the design will be **unsatisfactory from the point of wear or durability.**

For safer design

$$F_w \geq F_d$$

$$i.e., d_1\, b\, Q\, k \geq F_d$$

$$105 \times 100 \times 1.43 \times k \geq 19839.85$$

$$\therefore Load\ stress\ factor\ \ k \geq 1.32$$

$$i.e., \frac{{\sigma_{es}}^2\, Sin\ \propto}{1.4}\left[\frac{1}{E_1}+\frac{1}{E_2}\right] \geq 1.32$$

$$i.e., \frac{{\sigma_{es}}^2\, Sin\ 20}{1.4}\left[\frac{1}{2.02 \times 10^5} + \frac{1}{1.00 \times 10^5}\right] \geq 1.32$$

$$\therefore\ {\sigma_{es}}^2 \geq 361405.46$$

$$\therefore\ \sigma_{es} \geq 601 MPa$$

$$i.e., (2.75 \times BHN - 70) \geq 601$$

$$i.e., BHN \geq 244$$

OR

From the Table (T 12.16)(P-193) for $\alpha = 20^0$

$$k \geq 1.32\ MPa$$

Surface hardness no for Steel - Pinion = 250 BHN

Surface hardness no for C I - Gear = 180 BHN

Hence suggested average surface hardness for the gear pair $\geq 282.86\ BHN$

Hence if the average surface hardness of the gear pair is more than 282.86 BHN, then the design will be satisfactory from the stand point of wear or durability also.

6. A gear drive is required to transmit a maximum power of 22.5 kW. The velocity ratio is 2:1 and speed of the pinion is 200 rpm. The approximate center distance between the shafts may be taken as 600 mm. the tooth has 20^0 stub involutes profile. The static stress for the gears material (which is CI) may be taken as 60 MPa and face width as 10 times the module. Design the spur gear and check dynamic and wear loads. The deformation or dynamic factor May be taken as 80 and the material combination factor for wear as 1.4.

(June / July2011)(Marks 15)

Data:

$$\propto = 20^0\ Stub\ I\ system\ ; P = 22.5\ kW\ ;\ n_1 = 200\ rpm\ ;\ c = 600\ mm;$$

$$C = 80\ ;\ Pinion\ \&\ Gear\ material - Cast\ iron$$

allowable stress for $Pinion\ \&\ Gear$, $\sigma_{d_1} = \sigma_{d_2} = 60\ MPa$

Solution: *Velocity ratio (i)*

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_1}{d_2}$$

$$2 = \frac{200}{n_2}$$

$$n_2 = \frac{200}{2}$$

$$\therefore\ Speed\ of\ Gear\quad n_2 = 100\ rpm$$

$$Centre\ distance,\quad c = \frac{d_1 + d_2}{2} = \frac{d_1 + i\, d_1}{2} = \frac{d_1(1+i)}{2}$$

$$600 = \frac{d_1(1+2)}{2} = \frac{3 \times d_1}{2} \qquad \therefore (d_2 = i\, d_1)$$

$$d_1 = 400\ mm$$

$\therefore$ *Pitch circle diameter of Pinion* $\quad d_1 = 400\ mm$

$$\therefore d_2 = i\, d_1\ ; \quad d_2 = 2 \times 400$$

$\therefore$ *Pitch circle diameter of Gear* $\quad d_2 = 800\ mm$

To determine 'y' the lewis form factor

For 20^0 *Stub profile*

$$y = 0.175 - \frac{0.95}{Z} \quad for\ 14\frac{1}{2}^0\ FDI\ system.$$

$$(E - 12.17c)(P - 163) \quad (General\ equation)$$

Temporarily Assume $Z_1 = 20\ Teeths$

$$Z_1 = 20\ Teeths, \qquad \therefore Z_2 = 2 \times Z_1 = 2 \times 20 = 40\ Teeths$$

$$For\ Pinion\ y_1 = 0.175 - \frac{0.95}{Z_1}$$

$$y_1 = 0.175 - \frac{0.95}{20}$$

$$y_1 = 0.1275$$

$$For\ Gear \quad y_2 = 0.175 - \frac{0.95}{Z_2}$$

$$y_2 = 0.175 - \frac{0.95}{40}$$

$$y_2 = 0.15125$$

Identifying the weaker member

Particulars	σ_d	y	$\sigma_d\, y$	Remarks
Pinion	60	0.1275	7.65	**Weaker**
Gear	60	0.15125	9.075	

Since $\sigma_{d_2}\, y_2 < \sigma_{d_1}\, y_1$, *Gear is weaker member*.

Design should be based on Gear (weaker member).

Tangential tooth load

$$F_t = \frac{1000 \times P \times C_s}{v} \ \ldots\ldots (E\ 12.20\ a)(P - 164) \quad (General\ equation)$$

$$F_{t_1} = \frac{1000 \times P \times C_{s_1}}{v_1} \quad (Design\ equation)$$

Where, P = 2.25 kW= Power in kW.

$$C_s = Service\ factor\ form\ From\ (T\ 12.8)(P - 187)$$

Assume medium shock and $8 - 10$ *hours duty per day*

$$C_{s_1} = 1.5\ ;\ Speed = n_1 = 100\ rpm$$

$$v = Pitch\ line\ velocity\ of\ weaker\ member, \quad m/s$$

$$v = \frac{\pi\, d\, n}{60000} = \frac{\pi\,(m\, Z)\, n}{60000}, \quad m/s \ (\therefore d = m\, Z) \quad (General\ equation)$$

$$v_1 = \frac{\pi\, d_1\, n_1}{60000} = \frac{\pi \times 800 \times 100}{60000}$$

$$v_1 = 4.189 \quad m/s$$

$$\therefore \quad F_{t_1} = \frac{1000 \times 22.5 \times 1.5}{4.189}$$

$$F_{t_1} = 8056.8 \quad N \qquad \ldots\ldots\ldots Equn\ (i)$$

(ii) Tangential tooth load from lewis equation:

$F_t = \sigma_d \, C_v \, b \, y \, p \quad \dots\dots(E\ 12.15)(P\ 163) \quad (General\ equation)$

$F_{t_1} = \sigma_{d_1} \, C_{v_1} \, b \, y_1 \, p \quad (Design\ equation)$

Where $\sigma_{d_1} = \ allowable\ static\ stress\ of\ the\ pinion \ = 60\ MPa$

$\text{b} = \text{face width} \quad 9.5\text{m} \le \text{b} \le 12.5\ \text{m} \quad \dots\dots(E\ 12.18)(P\ 164)$

Take face width, $\text{b} = 10\ \text{m}$

$Circular\ Pitch, \quad p = \pi m$

$Velocity \quad v_1 = 4.189 \ {}^{m}/_{s}$

$Velocity\ factor\ [\ Barth's formula\ ]\ \ (C_v)$

$$C_v = \frac{3.05}{3.05 + v} \qquad for\ v \le 8\ {}^{m}/_{s}$$

$\dots\dots(E\ 12.19\ a)(P\ 164)\ (General\ equation)$

$$C_{v_1} = \frac{3.05}{3.05 + v_1} \qquad (Design\ equation)$$

$$C_{v_1} = \frac{3.05}{3.05 + 4.189}$$

$$C_{v_1} = 0.4213$$

$y_1 = \text{Lewis form factor of the Pinion}$

$$For\ Gear \quad y_1 = 0.175 - \frac{0.95}{Z_1}$$

$$y_1 = 0.175 - \frac{0.95}{\left(\frac{d_1}{m}\right)}$$

$$y_1 = 0.175 - \frac{0.95 \times m}{d_1}$$

$$y_1 = 0.175 - \frac{0.95 \times m}{400}$$

$$y_1 = 0.175 - 2.375 \times 10^{-3} m$$

$$F_{t_1} = 60 \times 0.4213 \times (10 \times m) \times y_1 \times (\pi \times m)$$

$$F_{t_1} = 794.13\, m^2 y_1 \quad N \qquad \dots\dots\dots\ Equn\ (ii)$$

Equating equation (*i*) & (*ii*)

$$794.13\, m^2 y_1 = 8056.8$$

$$m^2 y_1 \ge 10.145 \qquad \dots\dots\dots\ Equn\ (iii)$$

To find module 'm'

Trial 1:- *Let select ' m = 5'*

$$y_1 = 0.175 - 2.375 \times 10^{-3} m$$

$$y_1 = 0.175 - 2.375 \times 10^{-3} \times 5$$

$$y_1 = 0.163$$

Now from equation ..... (*iii)*

$$m^2 y_1 \ge 10.145 \qquad \dots\dots\dots\ Equn\ (iii)$$

$$5^2 \times 0.163 \ge 10.145$$

$$4.075 < 10.145 \qquad \therefore not\ suitable.$$

Trial 2:- *Let select ' m = 8'*

$$y_1 = 0.175 - 2.375 \times 10^{-3} m$$

$$y_1 = 0.175 - 2.375 \times 10^{-3} \times 8$$

$$y_1 = 0.156$$

Now from equation ..... (*iii)*

$$m^2 y_1 \geq 10.145 \qquad \ldots\ldots\ldots \; Equn\ (iii)$$

$$7^2 \times 0.156 \geq 10.145$$

$$9.984 < 10.145 \qquad \therefore not\ suitable.$$

Trial 3:- *Let select ' m = 9'*

$$y_1 = 0.175 - 2.375 \times 10^{-3} m$$

$$y_1 = 0.175 - 2.375 \times 10^{-3} \times 9$$

$$y_1 = 0.1536$$

Now from equation ..... (*iii)*

$$m^2 y_1 \geq 10.145 \qquad \ldots\ldots\ldots \; Equn\ (iii)$$

$$9^2 \times 0.1536 \geq 10.145$$

$$12.44 > 10.145 \qquad \therefore suitable.$$

$$\therefore Take \qquad m = 9\ mm$$

$y_1 = 0.1536;$ $\quad y_1 = 0.175 - \frac{0.95}{Z_1}$

$$\therefore \quad 0.1536 = 0.175 - \frac{0.95}{Z_1}$$

$$Z_1 = 46$$

∴ Face width 'b' b = 10 m = 10 x 9 = 90 mm.

Dimensions

Calculate all important geometric parameter of tooth profile by using the equations given in Table (T 12.4)(P 183)

Properties of involutes teeth

Tooth Characteristics		20^0 Stub	*Pinion*	*Gear*
Addendum	h_a	0.8 m	7	7
Dedendum	h_f	m	9	11
Thickness of the tooth	t	1.571 m	15	18
No of Teeth	z		44	88
Pitch circle diameter	d	m z	400	800

Check for the stress

Allowable stress by the equation

$$\sigma_{allow} = (\sigma_d\, C_v)_{allow} \qquad (General\ equation)$$

$$\sigma_{allow} = \left(\sigma_{d_1}\, C_{v_1}\right)_{allow} \qquad (Design\ equation)$$

$$\sigma_{allow} = 60 \times 0.413$$

$$\sigma_{allow} = 24.78\ MPa$$

Induced stress

$$\sigma_{ind} = (\sigma_d\, C_v)_{ind} = \frac{F_t}{b\, y\, p}$$

$$\dots\dots(E\ 12.15)(P\ 163) \qquad (General\ equation)$$

$$\sigma_{ind} = (\sigma_d\, C_v)_{ind} = \frac{F_{t_1}}{b\, y_1\, p} \qquad (Design\ equation)$$

$$\sigma_{ind} = \frac{8056.8}{90 \times 0.1536 \times \pi \times 9}$$

$$\sigma_{ind} = 20.61\ MPa$$

If $\sigma_{ind} < \sigma_{allow} (20.61\ MPa < 24.78\ MPa)$, then the design is satisfactory.

Checking for dynamic load

Acc to bucking ham's equation

Dynamic load

$$F_d = F_t + \frac{K_3\, v\,(C\, b + F_t)}{K_3\, v + \sqrt{(C\, b + F_t)}} \quad \dots\dots(E\ 12.33)(P\ 166) \ (General\ equation)$$

$$F_d = F_{t_1} + \frac{K_3\, v_1\left(C\, b + F_{t_1}\right)}{K_3\, v_1 + \sqrt{\left(C\, b + F_{t_1}\right)}} \qquad (Design\ equation)$$

The deformation or dynamic factor, $C = 80\ MPa$

Where, $F_{t_1} = 8056.8\ N$; $K_3 = 20.67$; $v_1 = 4.189\ ^{m}/_{s}$;

C = 80 MPa; b = 90 mm;

$$F_d = 8056.8 + \frac{20.67 \times 4.189 \times (80 \times 90 + 8056.8)}{20.67 \times 4.189 + \sqrt{(80 \times 90 + 8056.8)}}$$

$$F_d = 8056.8 + \frac{1321034.9}{210.1}$$

$$F_d = 8056.8 + 6287.65$$

$$F_d = 14344.3\ N$$

Check for endurance strength$(\boldsymbol{F_{en}})$

$$F_{en} = \sigma_{en} b\, Y\, m \quad \dots\dots(E\ 12.34)(P\ 166) \qquad (General\ equation)$$

$$F_{en_1} = \sigma_{en_1} b\, Y_1\, m \qquad (Design\ \ equation)$$

In lewis equation the allowable bending stress is approximately $\frac{1}{3}$ of ultimate stress.

∴ ultimate stress of the pinion material, $\sigma_u = 3 \times \sigma_{d_1} = 3 \times 60 = 180\ MPa$

Now endurance limit $\sigma_{en} = 0.5 \times \sigma_u = 0.5 \times 180 = 90\ MPa$

Where, $\sigma_{en} = 90\ MPa$; $b = 90\ mm$; $Y_1 = \pi\, y_1 = \pi \times 0.1536 = 0.4825\ ^{m}/_{s}$; m = 9 mm.

$$F_{en_1} = 90 \times 90 \times 0.4825 \times 9$$

$$F_{en_1} = 35177.8\ N$$

Since $F_{en} > F_d$ $(35177.8\ N > 14344.3\ N)$, the design will be ***satisfactory from the point of wear or durability.***

Check for wear load

Acc to bucking ham's equation wear load

$$F_w = d_1\, b\, Q\, k \quad \ldots\ldots (E\ 12.36\ a)(P\ 167) \quad (General\ equation)$$

$$F_w = d_1\, b\, Q\, k \quad (Design\ equation)$$

Where ratio factor

$$Q = \frac{2\, d_2}{d_2 + d_1} = \frac{2\, z_2}{z_2 + z_1} \quad \ldots\ldots (E\ 12.36\ c)(P\ 167)$$

$$Q = \frac{2\, d_2}{d_2 + d_1} = \frac{2 \times 800}{800 + 400}$$

$$Q = 1.33$$

$$k = 1.4\ MPa$$

i.e., $F_w = d_1\, b\, Q\, k = 400 \times 90 \times 1.33 \times 1.4$

$$F_w = 67183.2\ N$$

Since $F_d < F_w$ $(14344.3\ N < 67183.2\ N)$

From the Table (T 12.16)(P- 193) for $\alpha = 20^0\ Stub$

$$k = 1.4\ MPa$$

Surface hardness no for C I - Pinion = 250 BHN

Surface hardness no for C I - Gear = 180 BHN

The design will be **satisfactory from the point of wear or durability.**