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06MAT41

Fourth Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, selecting at least TWO questions from each part.*
2. Use of statistical tables is permitted.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Find the y(0.1) correct to 6 decimal places by Taylor series method when $dy/dx = xy + 1$, $y(0) = 1.0$. (Consider upto 4^{th} degree term). **(06 Marks)**

b. Using Runge-Kutta method of order 4, compute y(0.2) for the equation, $y' = y - \frac{2x}{y}$, $y(0) = 1.0$ (Take h = 0.2). **(07 Marks)**

c. Given that $y' = x^2(1+y)$ and $y(1) = 1.0$, $y(1.1) = 1.233$, $y(1.2) = 1.548$ and $y(1.3) = 1.979$, compute y(1.4) by Adams-Bashforth method. Apply correct formula twice. **(07 Marks)**

2 a. Show that Z^n is analytic. Hence find its derivative. **(06 Marks)**

b. Find a bilinear transformation which maps the points 0, 1, i in the Z-plane onto 1 + i, –i, 2 – i in the W plane. **(07 Marks)**

c. Find the analytic function u + iv, where u is given to be $u = e^x[(x^2 - y^2)\cos y - 2xy \sin y]$. **(07 Marks)**

3 a. Derive Couchy's integral formula in the form

$$f(a) = \frac{1}{2\pi i}\int_c \frac{f(z)dz}{z-a}$$ **(06 Marks)**

b. Expand $f(z) = \frac{7z^2 + 9z - 18}{z^3 - 9z}$ in the Laurent series that is valid for
i) $|z| > 3$ ii) $0 < |z-3| < 3$. **(07 Marks)**

c. Evaluate $\int_c \tan z\, dz$, where c is $|z| = 2.5$ **(07 Marks)**

4 a. Find the series solution of $\frac{d^2y}{dx^2} + xy = 0$. **(06 Marks)**

b. Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. **(07 Marks)**

c. Reduce the differential equation $x\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + k^2xy = 0$ to Bessel's equation. Obtain the solution. **(07 Marks)**

PART – B

5 a. Fit a curve of the form $y = ab^x$ for the data given below: (06 Marks)

x :	2	4	6	8	10	12
y :	1.8	1.5	1.4	1.1	1.1	0.9

b. Find the coefficient of correlation for the following data: (07 Marks)

x :	55	56	58	59	60	60	62
y :	35	38	39	38	44	43	45

c. In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body.

i) What is the probability that mathematics is being studied?

ii) If a student is selected a random and is found to be studying mathematics, find the probability that the student is a girl. (07 Marks)

6 a. Suppose a random variable X takes the values –3, –1, 2 and 5 with respective probabilities $\frac{2k-3}{10}, \frac{k-2}{10}, \frac{k-1}{10}, \frac{k+1}{10}$. Find the value of k and i) find $P[-3 < X < 4]$ and ii) $P[X \le 2]$. (06 Marks)

b. Suppose that the student IQ scores form a normal distribution with mean 100 and standard deviation 20. Find the percentage of students whose i) score is less than 80 ii) score falls between 90 and 140, iii) Score more than 120. (07 Marks)

c. Obtain mean and variance of binomial distribution function. (07 Marks)

7 a. A sample of 1000 days is taken from meteorological records of a certain district and 120 of them are found to be foggy. What are the probable 99% confidence limits to the proportion of foggy days in the district? (06 Marks)

b. The following table gives the number of bus accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week, using χ^2 test. (07 Marks)

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	16	8	12	11	9	14	84

c. The life X of certain computer is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the hypothesis that $\mu = 800$ hours against the alternate hypothesis $\mu \neq 800$ hours at i) 0.5% and 1% level of significance. (07 Marks)

8 a. A fair coin is tossed 4 times. Let X denote the number of heads occurring and let Y denote the longest string of heads occurring. Find the joint distribution function of X and Y. (06 Marks)

b. A man's gambling luck follows a pattern. If he wins a game the probability of winning the next game is 0.6. However, if he loses a game, the probability of losing the next game is 0.7. There is an even chance that he wins the first game.

i) Find the transition matrix of the Markov process. ii) Find the probability that he wins the third game. iii) Find out how often, in the long run, he wins. (07 Marks)

c. Explain: i) Transient state ii) Absorbing state and iii) Recurrent state by means of an example each. (07 Marks)

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