NEW SCHEME

CS42

USN | | | | | | | | | | |

Fourth Semester B.E. Degree Examination, July/August 2004

Computer Science /Information Science and Engineering

Graph Theory and Combinatorics

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions, choosing atleast TWO full questions from each Part.*

PART - A

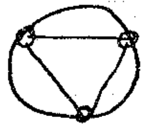
1. (a) Show that there is no graph with 12 vertices and 28 edges where,
i) the degree of each vertex is either 3 or 4
ii) The degree of each vertex is either 3 or 6. (6 Marks)

(b) How many edge disjoint Hamiltonian cycles exist in the complete graph on seven vertices ? Also draw the graph to show these Hamiltonian cycles. (7 Marks)

(c) Define isomorphism of two graphs. Give an example to show that two graphs need not be isomorphic though they have equal number of edges, equal number of vertices and equal number of vertices with a given degree sequence. (7 Marks)

2. (a) Define complete bipartite graph. Prove that Kuratowski's second graph, $K_{3,3}$ is nonplanar. (7 Marks)

(b) Draw the geometric dual of the graph given in the following figure. (6 Marks)



(c) Prove that the vertices of every planar graph can be properly coloured with five vertices. (7 Marks)

3. (a) Prove that a tree with n vertices has $n-1$ edges. (6 Marks)

(b) i) Define prefix code

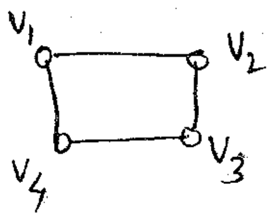
ii) Which of the following sets represent prefix code?

State reasons

$A = \{000, 001, 01, 10, 11\}$

$B = \{1, 00, 01, 000, 0001\}$ (7 Marks)

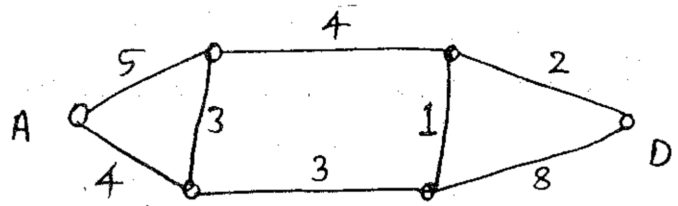
(c) Find all the spanning trees of the graph given below. (7 Marks)



4. (a) Explain Prim's algorithm for finding shortest spanning tree of a weighted graph. (7 Marks)

   (b) Define i) Cutset, ii) Edge connectivity iii) Vertex connectivity with one example each (6 Marks)

   (c) Find the maximum flow possible between the vertices A and D for the following graph. (7 Marks)



PART - B

5. (a) In how many ways can one distribute 10 identical white marbles among six distinct containers ? (6 Marks)

   (b) A message is made up of 12 different symbols and is to be transmitted through a communication channel. In addition to the 12 symbols the transmitter will also send a total of 45 (blank) spaces between the symbols with atleast three spaces between each pair of consecutive symbols. In how many ways can transmitter send such a message ? (7 Marks)

   (c) Use Catlan numbers to find the number of ways to list eight symbols which include four 0's and four 1's so that in each case the number of 0's never exceed number of 1's. (7 Marks)

6. (a) State and prove extended Pigenhole principle. Hence show that if 30 dictionaries in a library contain a total of 61, 327 pages, then one of the dictionaries must have atleast 2045 pages. (6 Marks)

   (b) Using the principle of inclusion and exclusion determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2 or 3 or 5. (7 Marks)

   (c) Define derangements. Find the number of derangements of 1, 2, 3, 4 using exponential series technique. Also made a list of all the derangements of the above example mentioned. (7 Marks)

7. (a) Find the exponential generating function of the sequence $1, 2, 2^2, 2^3, 2^4, \ldots$ (7 Marks)

   (b) Find the number of partitions of positive integer $n = 6$ in to distinct summands as a coefficient of x^6 in the generating funtion of $p_d(6)$. Also list these partitions. (7 Marks)

   (c) What is summation operator ? Explain. (6 Marks)

8. (a) Find the generating function of the linear recurrence relation

   $C_n = 3C_{n-1} - 2C_{n-2}$ with $C_1 = 5, C_2 = 3.$ (6 Marks)

   (b) Find the generating function of

   $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 2$, $a_0 = -1$ & $a_1 = 8$ (7 Marks)

   (c) Find the generating function of

   $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$ (7 Marks)

** * **

NEW SCHEME

CS42

USN

Fourth Semester B.E. Degree Examination, January/February 2005

Computer Science and Information Science Engineering

Graph Theory & Combinations

Time: 3 hrs.] [Max.Marks : 100

Note: ***1. Answer FIVE full questions choosing atleast TWO full question from each Part.***
2. All questions carry equal marks.

PART - A

1. (a) Define :

i) Connected graph

ii) Spanning subgraph and

iii) Compliment of a graph. Give one example for each. **(6 Marks)**

(b) Explain with example graph isomorphism. Show that in a graph G the number of odd degree vertices is even. **(7 Marks)**

(c) Write a note on "Konigsberg-bridge" problem. **(7 Marks)**

2. (a) Define :

i) Planar graph

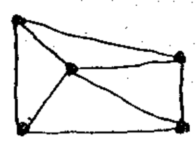
ii) Complete Bipartite graph and

iii) Dual of a planar graph.

Give one example for each. **(6 Marks)**

(b) Show that in any connected planar graph with n vertices, e-edges and f-faces $e-n+2=f$. (Eulers formula). **(7 Marks)**

(c) Define chromatic number and chromatic polynomial. Find the chromatic polynomial for the graph given below. **(7 Marks)**



3. (a) Define :

Tree ii) Binary rooted tree and iii) Prefix code.

Give one example for each. **(6 Marks)**

(b) Prove that a tree with n vertices has $n-1$ edges. **(7 Marks)**

(c) Obtain a prefix code to send the message ROAD IS GOOD using labeled Binary tree and hence encode the message. **(7 Marks)**

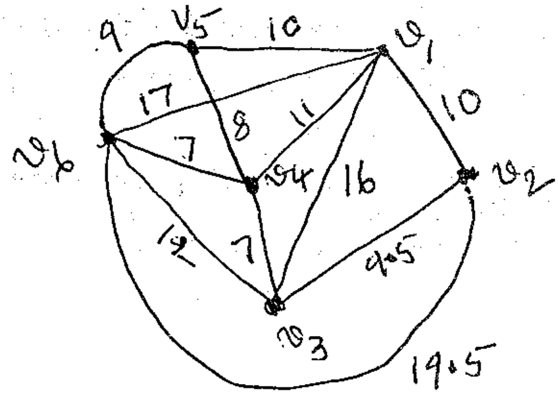
4. (a) Define : **(7 Marks)**

i) Vertex connectivity ii) Edge connectivity.
iii) Bridge iv) Cut vertex with an example.

Contd.... 2

(b) Prove that the maximum flow possible between two vertices a and b in a network (graph) is equal to the minimum of the capacities of all cut-sets with respect to a and b. **(6 Marks)**

(c) Find the shortest spanning tree using Prims algorithm for the weighted graph given below. **(7 Marks)**



PART - B

5. (a) State sum and product rule of counting. Give one example for each. **(6 Marks)**

(b) i) How many 9 letter words can be formed using the letters of the word "Difficult"? **(7 Marks)**

ii) A certain question paper contains two parts A and B each contains 4 questions. How may different ways a student can answer 5 questions by selecting atleast two questions from each part?

(c) In how many ways can one travel in the xy-plane from (0,0) to (3,3) using the moves $R : (xy) \to (x + 1, y)$ and $U : (x, y) \to (x, y + 1)$ if the path taken may touch but never rise above the line $y = x$? Draw two such paths in xy-plane. **(7 Marks)**

6. (a) State Pigen hole principle and generalised Pigen hole principle. Show that if any five numbers from 1 to 8 are choosen then two of them will add up to 9. **(6 Marks)**

(b) Define Rook polynomials and Forbidden positions. There are 6 pairs of students gloves in a box and each pair is of different colour. Suppose right gloves are distributed at random to six students. and then the left gloves are distributed at random to them. Find the probability that

i) No student gets a matching pair

ii) Everybody gets a matching pair. **(8 Marks)**

(c) In a Dormitory there are 12 students who take art course (A), 20 who take biology (B), 20 who take chemistry (C) and 8 who take drama course (D). There are 5 students for both A and B, 7 students for both A and C, 4 students for A and D, 16 students for B and C, 4 students for B and D and 3 students for C and D. There are 3 students who take A, B and C, 2 for A, B and D, 2 for B, C and D, 3 for A, C and D. Finally there are 2 in full four courses. It is also known that there are 71 students in the dormitory who have not signed up for any of these courses. Find the total number of students in the dormitory. **(6 Marks)**

7. (a) Define (ordinary) generating function and exponential generating function. Give one example for each. **(6 Marks)**

(b) Find the coefficient of x^{18} in the product

$(x + x^2 + x^3 + x^4 + x^5)\ (x^2 + x^3 + x^4 + - - - -)^5$ **(7 Marks)**

(c) Find :

i) The sequence corresponding to the generating function $3x^3 + e^{2x}$ **(7 Marks)**

ii) Generating function for the sequence 0,2,6,12,20,30,42 - - - .

8. (a) Solve the recurrence relation (Fibonacci relation) $F_{n+2} = F_{n+1} + F_n$ given $F_o = 0$ & $F_1 = 1$ and $n \geq 0$ **(6 Marks)**

(b) Using generating function solve :

$y_{n+2} - 4y_{n+1} + 3y_n = 0$ given that $y_0 = 2, \quad y_1 = 4$ **(7 Marks)**

(c) Find the general solution of $s(k) - 3\ s(k-1) - 4\ s(k-2) = 4^k$ **(7 Marks)**

** * **

NEW SCHEME

CS42

USN

Fourth Semester B.E. Degree Examination, July/August 2005

Computer Science and Information Science Engineering

Graph Theory & Combinatorics

Time: 3 hrs.] [Max.Marks : 100

Note: ***Answer FIVE full questions choosing atleast TWO full questions from each Part.***

PART - A

1. (a) Define a directed garph and an undirected graph.

i) If $G = (V, E)$ is an undirected graph with $|V| = v$, $|E| = e$, and no loops, prove that $2e \leq v^2 - v$.

ii) State the corresponding inequality for the case when G is directed. **(6 Marks)**

(b) For the undirected graph in fig 1(b), find and solve a recurrence relation for the number of closed $u - v$ walks of length $n \geq 1$.

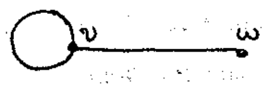


Fig. 1 (b) **(4 Marks)**

(c) Define an induced subgraph.

i) Let $G = (V, E)$ be an undirected graph, with $G_1 = (V_1, E_1)$ a subgraph of G. Under what condition(s) is G_1 not an induced subgraph of G ?

ii) For the graph G in Fig 1(c), find a subgraph that is not an induced subgraph. **(5 Marks)**

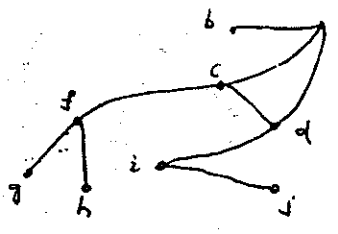


Fig 1 (c)

(d) Define graph isomorphism. Let $G = (V, E)$, $H = (V', E')$ be undirected graph with $f : V \to V'$ establishing isomorphism between the graphs.

i) Prove that $f^{-1} : V' \to V$ is also an isomorphism for G and H.

ii) If $a \in V$, prove that $deg(a)(inG)).\ deg(f(a))\ (inH)$ **(5 Marks)**

2. (a) Define a bipartite graph. Can a bipartite graph contain a cycle of odd length? Explan. **(6 Marks)**

(b) What is a Hamilton cycle ?

i) For $n \in Z^+$, $n \geq 2$, show that the number of distinct Hamilton cycles in the graph $K_{n,n}$ is $\frac{1}{2}(n-1)!\,n!$

ii) How many different Hamilton paths are there for $K_{n,n}$ & $n \geq 1$? **(7 Marks)**

(c) Consider the graph $K_{2,3}$ shown in fig 2(c), and let $\lambda \in Z^+$ denote the number of colours available to properly color the vertices of $K_{2,3}$.

i) How many proper colorings of $K_{2,3}$ have vertices a, b coloured the same ?

ii) How many proper colorings of $K_{2,3}$ have vertices a, b colored with different colors ?

iii) What is the chromatic polynomial of $K_{2,3}$? What is $\chi(K_{2,3})$?

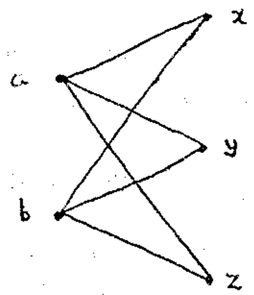


Fig. 2(c) **(7 Marks)**

3. (a) Define a tree. In every tree $T = (V, E)$, show that $|V| = |E| + 1$.

If a tree has four vertices of degree 2, one vertex of degree 3, two of degree 4, and one of degree 5, how many pendant vertices does it have?

(b) List the vertices in the tree shown in fig 3(b) when they are visited in a preorder traversal and in a post order traversal. **(6 Marks)**

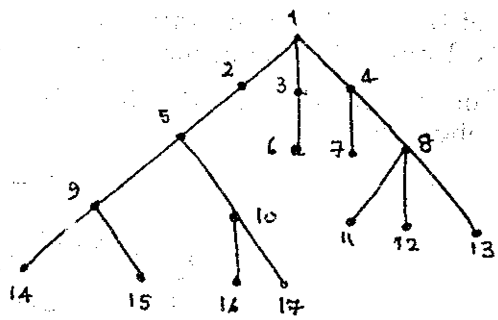
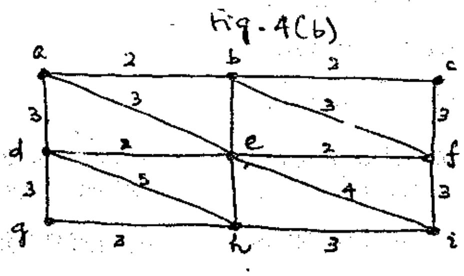


Fig. 3(b)

(c) Write Mergesort algorithm and derive its time complexity. **(6 Marks)**

4. (a) Write Kruskal's algorithm to find a minimal spanning tree for a loop-free undirected graph $G = (V, E)$ with $|V| = n$. Prove that the spanning tree obtained by this algorithm is optimal. Prove that the time-complexity of the algorithm is $O(n^2 log_2\, n)$. **(12 Marks)**

(b) Apply Kruskals algorithm to determine minimal spanning tree for the graph shown in Fig 4(b)



(8 Marks)

PART - B

5. (a) Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$

i) How many functions are there from A to B ? How many of these are one-to-one ? How many are onto ?

ii) How many functions are there from B to A ? How many of these are onto? How many are one-to-one ? **(10 Marks)**

(b) What are catalan numbers ? Consider the moves.

$R : (x, y) \rightarrow (x+1,\ y)$ and $U : (x, y) \rightarrow (x, y+1)$

In how many ways can one go

i) from $(0, 0)$ to $(6, 6)$ and not rise above the line $y = x$?

ii) from $(2, 1)$ to $(7, 6)$ and not rise above the line $y = x - 1$?

iii) from $(3, 8)$ to $(10, 15)$ and not rise above the line $y = x + 5$? **(4 Marks)**

(c) Let triangle ABC be equilateral, with $AB = 1$. Show that if we select 10 points in the interior of this triangle, there must be at least two whose distance apart is less than 1/3. (Hint : Use Pigeonhole principle) **(6 Marks)**

6. (a) i) In how many ways can the letters in ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters ? at least two pairs of consecutive identical letters ?

ii) Answer part (i), replacing two with three. **(8 Marks)**

(b) Give a combinatorial argument to verify that $\forall n \in Z^+$,

$$n! = \binom{n}{0} d_0 + \binom{n}{1} d_1 + \dots + \binom{n}{n} d_n = \sum_{k=0}^{n} \binom{n}{k} d_k,$$

(for each $1 \leq k \leq n$, d_k = the number of derangements of $1, 2, \dots k$; $d_0 = 1$) **(5 Marks)**

(c) A pair of dice, one red and the other green is rolled six times. We know that the ordered pairs $(1,1), (1,5), (2,4), (3,6), (4,2), (4,4), (5,1)$, and $(5,5)$ did not come up. What is the probability that every value come up on both the red die and the green one ? **(7 Marks)**

7. (a) For $n \in Z^+$, find in $(1 + x + x^2)(1 + x)^n$ the coefficient of

i) x^7 ii) x^8 iii) x^r for $0 \leq r \leq n + 2$, $r \in Z$ **(6 Marks)**

(b) Determine the generating function for the sequence a_0, a_1 ...... where a_n is the number of portions of the nonnegative integer n into

i) even summands

ii) distinct even summands

iii) distinct odd summands. **(6 Marks)**

(c) Define an exponential generating function (EGF). Find the EGF for the number of ways to arrange n letters, $n \geq 0$. selected from each of the following words :

i) HAWAII ii) MISSISSIPPI iii) ISOMORPHISM **(8 Marks)**

8. (a) The number of bacteria in a culture is 1000 (approximately), and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **(6 Marks)**

(b) Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$ and $a_0 = 1,\ a_1 = 4$. **(6 Marks)**

(c) Solve the following recurrence relations by the method of generating functions:

i) $a_{n+2} - 3a_{n+1} + 2a_n = 0,\ n \geq 0,\ a_0 = 1,\ a_1 = 6$

ii) $a_{n+2} - 2a_{n+1} + a_n = 2^n,\ n \geq 0,\ a_0 = 1,\ a_1 = 2$ **(8 Marks)**

** * **

NEW SCHEME

CS42

Reg. No.

Fourth Semester B.E. Degree Examination, January/February 2006

Computer Science and Information Science Engineering

Graph Theory & Combinatorics

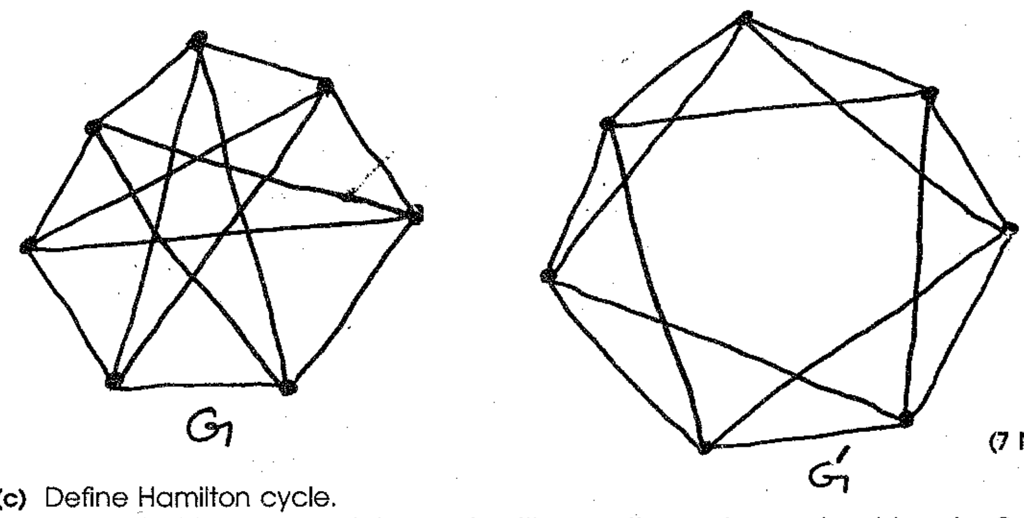
Time: 3 hrs.) (Max.Marks : 100

Note: ***Answer FIVE full questions choosing atleast TWO full questions from each Part.***

PART - A

1. (a) Determine $|V|$ for the following graphs G.
   i) G has nine edges and all vertices have degree 3.
   ii) G has 10 edges with two vertices of degree 4 and all others of degree 3. **(6 Marks)**

   (b) Define isomorphism of graphs. Show that the following two graphs are isomorphic.

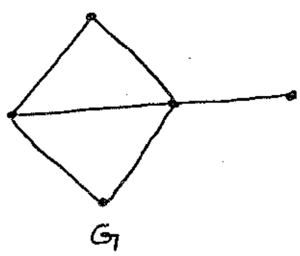


(7 Marks)

   (c) Define Hamilton cycle.
   Prove that in the complete graph with n vertices, where n is odd and ≥ 3, there are $\frac{n-1}{2}$ edge - disjoint Hamilton cycles. **(7 Marks)**

2. (a) Define i) a planar graph ii) a bipartite graph, and iii) a complete bipartite graph. Give one example for each **(6 Marks)**

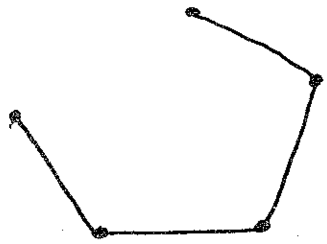
   (b) Find the geometric dual of the graph $G = (V, E)$. Write down any four observations of G and its dual.



(7 Marks)

Contd.... 2

(c) Define chromatic number of a graph.
Find the chromatic polynomial $P(G,\lambda)$ for the following graph G. Hence find the chromatic number.



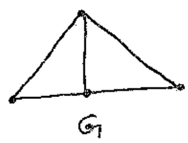
(7 Marks)

3. (a) If a tree $T=(V,E)$ has four vertices of degree 2, one vertex of degree 3, two vertices of degree 4, and one vertex of degree 5, find the number of pendant vertices in T. **(6 Marks)**

(b) Construct an optimal prefix code for the symbols

a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7, respectively. **(7 Marks)**

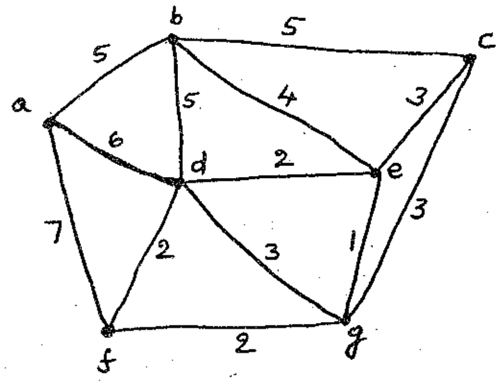
(c) Define spanning tree of a graph. Find all the spanning trees of the following graph.



(7 Marks)

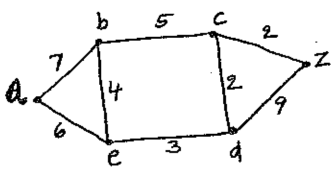
4. (a) Define i) Cutset, ii) Edge - connectivity, and iii) vertex connectivity. Give one example for each. **(6 Marks)**

(b) Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown below :



(7 Marks)

(c) For the network shown below, find the capacities of all the cutsets between the vertices a and Z, and hence determine the maximum flow between a and Z.



(7 Marks)

Part – B

PART - B

5. **(a)** How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000 ? **(6 Marks)**

(b) Consider the following program segment, where i, j and k are integer variables.

```
for i: = 1 to 20 do
  for j : = 1 to i do
    for k : = 1 to i do
         print (i * j + k)
```

How many times is the print statement executed in this program segment ? **(7 Marks)**

(c) Use catalar numbers to find in how many ways can one arrange four 1's and four -1's so that all eight partial sums (starting with the first summand) are non-negative? List all the arrangements. **(7 Marks)**

6. **(a)** State the Pigeonhole principle and the extended pigeonhole principle. Show that if any six numbers from 1 to 9 are chosen then two of them will add up to 10. **(6 Marks)**

(b) In a certain area of the countryside, there are five villages a, b, c, d, e. An engineer is to device a system of two-way roads so that, after the system is completed, no village will be isolated. In how many ways can he do this ? **(7 Marks)**

(c) Define derangement. In how many ways we can arrange the numbers 1, 2, 3, ... 10 so that 1 is not in 1^{st} place, 2 is not in 2^{nd} place and so on and 10 is not in 10^{th} place. **(7 Marks)**

7. **(a)** Determine the generating function of the numeric function

$$a_r = \begin{cases} 2^r & if\ r\ is\ even \\ -2^r & if\ r\ is\ odd \end{cases}$$ **(6 Marks)**

(b) Use generating function to determine how many four element subsets of $S = \{1, 2, 3, ...15\}$ contain no consecutive integers. **(7 Marks)**

(c) A company hires 11 new employees, each of whom is to be assigned to one of four subdivisions. Each subdivision will get at least one new employee. In how many ways can these assignments be made ? **(7 Marks)**

8. **(a)** The number of virus affected files in a system is 1,000 and this increases 250 % every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(6 Marks)**

(b) Solve the recurrence relation.

$a_n - 6a_{n-1} + 9a_{n-2} = 0, \quad n \geq 2$
given $a_0 = 5, a_1 = 12$. **(7 Marks)**

(c) Find the generating function for the recurrence relation

$a_{n+2} - 5a_{n+1} + 6a_n = 2, \quad n \geq 0$ and $a_0 = 3, \quad a_1 = 7$. Hence solve it. **(7 Marks)**

** * **

CS42

USN

NEW SCHEME

Fourth Semester B.E. Degree Examination, July 2006
CS / IS
Graph Theory and Combinatonics

Time: 3 hrs.] [Max. Marks:100

Note: ***1. Answer any FIVE questions choosing at least TWO full questions from each of the parts A and B.***

PART - A

1 a. i) Determine $|V|$, given that $G=(V,E)$ is regular with 15 edges.

ii) Let $G=(V,E)$ be a connected undirected graph. What is the largest possible value for $|V|$ if $|E|=19$ and $\deg(v)\geq 4$ for all $v \in V$? **(07 Marks)**

b. Define isomorphism of graphs. Show that the following graphs are isomorphic. **(07 Marks)**

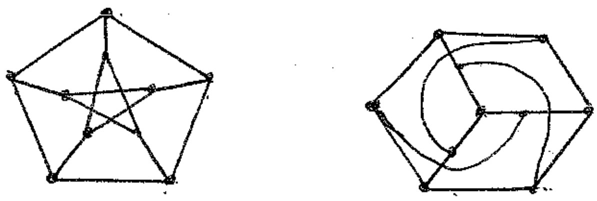


Fig. 1(b)

c. Define :
   i) Euler circuit
   ii) Hamilton cycle and
   iii) Hamilton path.

Give one examples for each. **(06 Marks)**

2 a. Define a planar graph. Show that the complete graph K_5 (Kuratowski's first graph) is non planar. **(07 Marks)**

b. Find the geometric dual of the following graph. Write down any four observations of the graph Fig. 2(b) and its dual. **(07 Marks)**

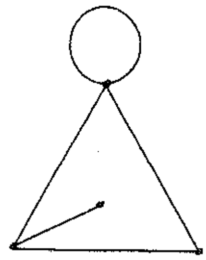


Fig. 2(b)

c. Define chromatic number of a graph. Find the chromatic polynomial $P(G,\lambda)$, where G is a cycle of length four. Hence find the chromatic number. **(06 Marks)**

3 a. Define a tree. Prove that, if $G=(V,E)$ is an undirected graph, then G is connected if and only if G has a spanning tree. **(07 Marks)**

b. Construct an optimal prefix code for the symbols a, b, c, d, e, f, g, h, i, j that occur (in a given sample) with respective frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3. **(07 Marks)**

c. Define :

i) Rooted tree

ii) Balanced tree and

iii) Prefix code.

Give one example for each. **(06 Marks)**

4 a. Define :

i) cutset

ii) Bridge

iii) Edge connectivity and

iv) Vertex connectivity.

Give one example for each. **(07 Marks)**

b. State Kruskal's algorithm. Using Kruskal's algorithm, find a minimal spanning tree for the weighed graph shown in Fig. 4(b).

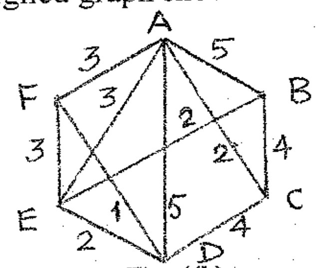


Fig. 4(b)

c. For the network shown in Fig. 4(c), find the capacities of all the cutsets between the vertices a and z and hence determine the maximum flow between a and z. **(06 Marks)**

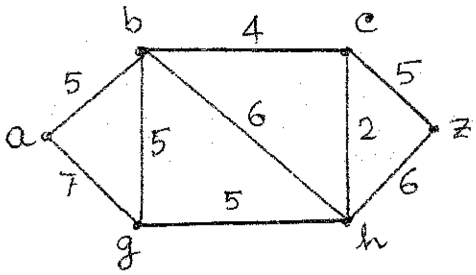


Fig. 4(c)

PART - B

5 a. A computer science professor has seven different programming books on a bookshelf. Three of the books deal with C^{++}, the other four with Java. In how many ways can the professor arrange these books on the shelf

i) If there are no restrictions.

ii) If the languages should alternate.

iii) If all the C^{++} books must be next to each other.

iv) If all the C^{++} books must be next to each other and all the Java books must be next to each other. **(07 Marks)**

Contd.... 3

3 a. Define a tree. Prove that, if $G = (V, E)$ is an undirected graph, then G is connected if and only if G has a spanning tree. **(07 Marks)**

b. Construct an optimal prefix code for the symbols a, b, c, d, e, f, g, h, i, j that occur (in a given sample) with respective frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3. **(07 Marks)**

c. Define :

i) Rooted tree
ii) Balanced tree and
iii) Prefix code.

Give one example for each. **(06 Marks)**

4 a. Define :

i) cutset
ii) Bridge
iii) Edge connectivity and
iv) Vertex connectivity.

Give one example for each. **(07 Marks)**

b. State Kruskal's algorithm. Using Kruskal's algorithm, find a minimal spanning tree for the weighed graph shown in Fig. 4(b).

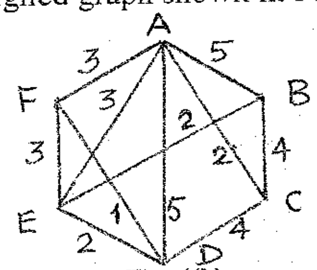


Fig. 4(b)

c. For the network shown in Fig. 4(c), find the capacities of all the cutsets between the vertices a and z and hence determine the maximum flow between a and z. **(06 Marks)**

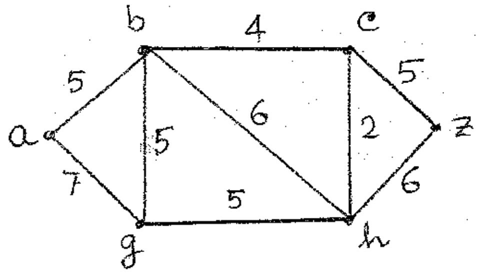


Fig. 4(c)

PART - B

5 a. A computer science professor has seven different programming books on a bookshelf. Three of the books deal with C^{++}, the other four with Java. In how many ways can the professor arrange these books on the shelf

i) If there are no restrictions.
ii) If the languages should alternate.
iii) If all the C^{++} books must be next to each other.
iv) If all the C^{++} books must be next to each other and all the Java books must be next to each other. **(07 Marks)**

CS42

USN

NEW SCHEME

Fourth Semester B.E. Degree Examination, July 2007
CS / IS
Graph Theory and Combinatorics

Time: 3 hrs.] [Max. Marks:100

Note : *Answer any FIVE full questions, choosing atleast two questions from each part.*

PART A

1 a. Let G be the following graph shown in fig.1(a). Then

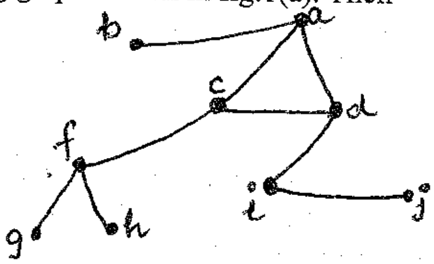


Fig.1(a)

i) How many connected subgraphs of G have four vertices and include a cycle? Also write these subgraphs.

ii) How many connected spanning subgraphs are there? Also write these subgraphs (any two). **(07 Marks)**

b. Define: i) Isomorphism of graphs ii) Ring sum of graphs iii) Circuit and cycle of a graph. **(06 Marks)**

c. If G = (V, E) is a loop-free undirected graph with $|V| = n \geq 3$, and if $|E| \geq \binom{n-1}{2} + 2$, then prove that G has a Hamiltonian cycle. **(07 Marks)**

2 a. Let G = (V, E) be a connected planar graph or multigraph with $|V| = v$ and $|E| = e$. Let 'r' be the number of regions in the plane determined by a planar embedding of G. Then show that $v - e + r = 2$. **(07 Marks)**

b. Define: i) Dual of a planar graph ii) Chromatic number iii) Complete bipartite graph. **(06 Marks)**

c. Let G be an undirected graph with subgraphs G_1, G_2. If $G = G_1 \cup G_2$ and $G_1 \cap G_2 = K_n$, for some $n \in Z^+$, then prove the following with usual notations:

$P(G,\lambda) = [P(G_1,\lambda).P(G_2,\lambda)]/\lambda^{(n)}$

Use the above result to find $P(G, \lambda)$ for the graph shown in fig.2(c). **(07 Marks)**

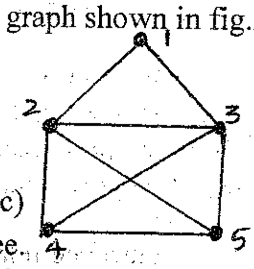


Fig.2(c)

3 a. i) Define m-ary tree and complete m-ary tree.

ii) How many internal vertices does a complete 5-ary tree with 817 leaves have? **(07 Marks)**

b. Obtain an optimal prefix code for the message FALL OF THE WALL. Indicate the code. **(06 Marks)**

c. Apply merge-sort to the list: -1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3. **(07 Marks)**

Contd.... 2

4 a. Apply Dijkstra's algorithm to the following weighted graph shown in fig.4(a) and determine the shortest distance from vertex 'a' to each of the other six vertices in the graph. **(10 Marks)**

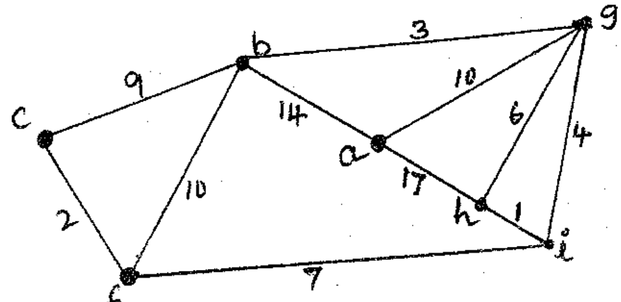


Fig.4(a)

b. i) Define matching and complete matching (with example).
ii) Define edge-connectivity and vertex-connectivity. Give an example for each. **(10 Marks)**

PART B

5 a. A woman has eleven close relatives and she wishes to invite five of them to dinner. In how many ways can she invite them in the following situations:
i) There is no restriction on the choice
ii) Two particular persons will not attend separately
iii) Two particular persons will not attend together. **(07 Marks)**

b. i) Find the coefficient of xyz^2 in the expansion of $(2x - y - z)^4$.
ii) Find the number of integer solutions of
$$x_1 + x_2 + x_3 + x_4 + x_5 = 30,$$
where $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0$. **(06 Marks)**

c. Define: i) Ramsey numbers ii) Stirling number of the second kind iii) The Pigeonhole principle. **(07 Marks)**

6 a. In how many ways can one arrange the letters in CORRESPONDENTS so that:
i) There are exactly two pairs of consecutive identical letters.
ii) There are atleast three pair of consecutive identical letters. **(07 Marks)**

b. In how many ways can the integers 1, 2, 3....., 10 be arranged in a line so that no even integer is in its natural place? **(06 Marks)**

c. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish to have apple, the boy B_3 does not want banana or mango and B_4 returns orange. In how many ways the distribution can be made so that no boy is displeased? **(07 Marks)**

7 a. Find a formula for the convolution of each of the following pairs of sequences:

i) $a_n = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & n \geq 5 \end{cases}$ and $b_n = n$, for all $n \in N$.

ii) $a_n = b_n = (-1)^n$, for all $n \in N$. **(07 Marks)**

b. In how many ways can we distribute 24 pencils to 4 children so that each child gets atleast 4 pencils but no more than nine? **(06 Marks)**

c. Find the number of ways in which 5 of the letters in "ENGINE" be arranged. **(07 Marks)**

8 a. Find and solve a recurrence relation for the number of binary sequences of length n that has no consecutive 0's. **(07 Marks)**

b. Solve the recurrence relation: $a_{n+1} + 3a_{n+1} + 2a_n = 3^n, \quad n \geq 0$,
given $a_0 = 0,\ a_1 = 0$ **(06 Marks)**

c. Solve the recurrence relation by the method of generating functions:
$a_{n+1} - a_n = n^2,\ n \geq 0, \qquad$ given $a_0 = 1$. **(07 Marks)**

*****

USN

CS42

Fourth Semester B.E. Degree Examination, Dec. 07 / Jan. 08

Graph Theory and Combinatorics

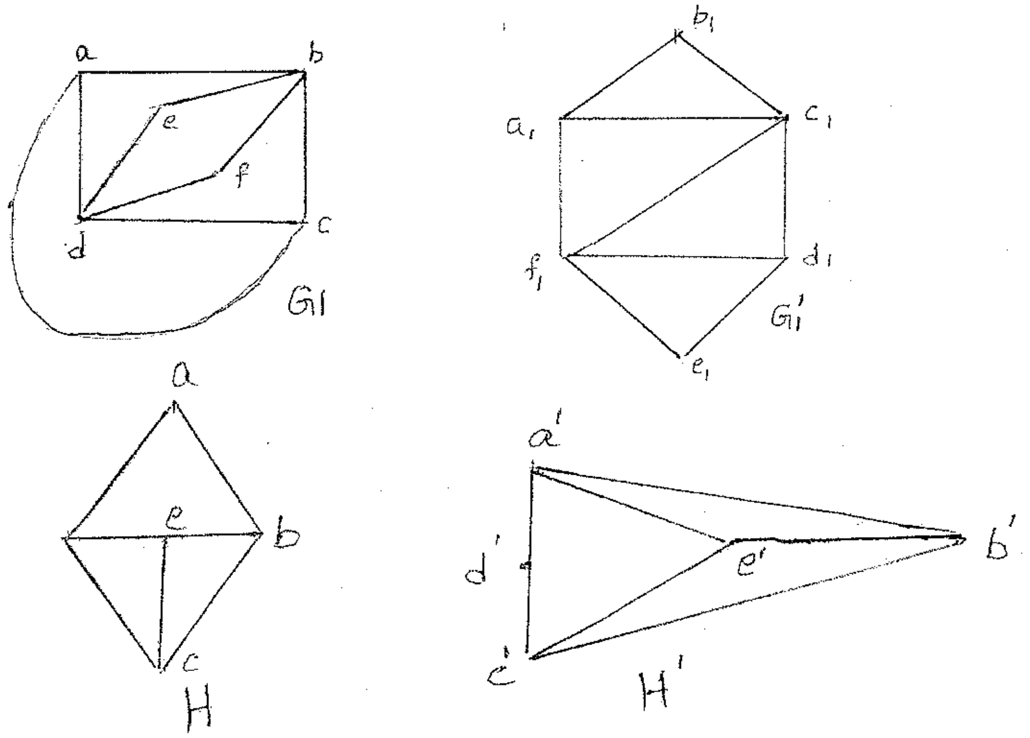
Time: 3 hrs. Max. Marks:100

Note : ***Answer FIVE full questions, choosing at least TWO from each PART.***

PART A

1 a. Show that if a bipatriate graph (X, Y, E) is regular, both X and Y have the same number of elements. **(06 Marks)**

b. State whether the following graphs are isomorphic or not. Justify your answer. **(08 Marks)**



c. Show that K_2, K_3 and K_4 are planar. **(06 Marks)**

2 a. If a graph of n vertices is isomorphic to its complement, then how many of vertices it must have? **(05 Marks)**

b. Consider a graph of even order P which has two components which are complete. Prove that minimum number of lines $= \frac{P(P-2)}{4}$. **(08 Marks)**

c. If G is a non directed graph of order 9, such that each vertex has degree 5 or 6, then prove pthat G has at least 5 vertices each of degree 6 or 6 vertices each of degree 5. **(07 Marks)**

3 a. Show the Euler circuit in the graph shown : **(08 Marks)**

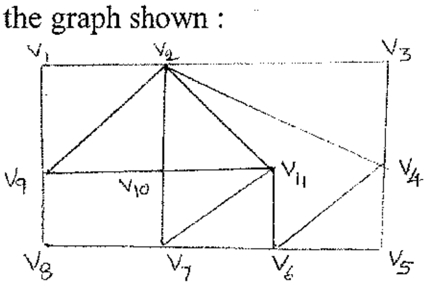


Fig. 3(a)

b. Find the chromatic polynomial for the graph. **(08 Marks)**

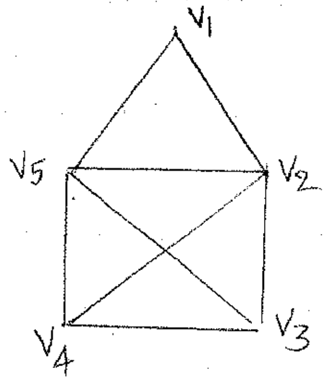


Fig. 3(b)

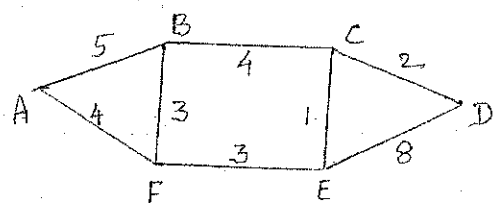


Fig. 3(c)

c. For the network shown, find the maximum flow between the vertices A and D, using the cut set of minimum capacity. **(04 Marks)**

4 a. Show that in a tree, if the degree of every non-pendant vertex is 3, the number of vertices in the tree is even. **(06 Marks)**

b. Find all the spanning trees of the graph given below : **(06 Marks)**

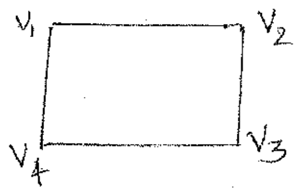


Fig. 4(b)

c. Three boys b_1, b_2, b_3 and four girls g_1, g_2, g_3, g_4 are such that
b_1 is a cousin of g_1, g_3, g_4,
b_2 is a cousin of g_2 and g_4,
b_3 is a cousin of g_2 and g_3.
If a boy can marry a cousin girl, find possible sets of such coupler. **(08 Marks)**

PART B

5 a. Find the number of proper divisors of 441,000. **(08 Marks)**

b. Find the number of integers $x \le 10^8$ so that the sum of the digits in x equals 31. **(08 Marks)**

c. Find the coefficient of $x^{11}y^4$ in the expansion of $(2x^3 - 3xy^2 + Z^2)^6$. **(04 Marks)**

6 a. State recurrence relation for fibonacci series and solve. **(08 Marks)**

b. Show $(n!)^2 > n^n$ for $n > 2$. **(08 Marks)**

c. A bag has few red marbles, white marbles, and blue marbles. What is the least number of marbles one should take out to be sure of getting at least 6 marbles of same color? **(04 Marks)**

7 a. State Catalan number generating sequence and find first five Catalan numbers. **(08 Marks)**

b. What is Ramsey number? Find R(4, 3). **(06 Marks)**

c. What is Bell number? Find B_5 interms of stirling number. **(06 Marks)**

8 a. Solve the recurrence relation

$$a_n = 2a_{n/2} + n - 1 \qquad n \ge 2 \qquad a_1 = 0 \qquad n = 2^K \text{ where } K \ge 1.$$ **(08 Marks)**

b. Show that 97 is the 25^{th} prime number. **(06 Marks)**

c. Show that any set of seven distinct integers includes two integers x and y such that at least one of x + y or x – y is divisible by 10. **(06 Marks)**

*****

USN | | | | | | | | | | |

06CS42

Fourth Semester B.E. Degree Examination, June-July 2009
Graph Theory & Combinatorics

Time: 3 hrs.

Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Determine $|\Upsilon|$ for the following graphs:
   i) G is regular with 15 edges.
   ii) G has 10 edges with two vertices of degree 4 and all others of degree 3. **(05 Marks)**

b. Define isomorphism of graphs. Show that the following graphs are isomorphic.

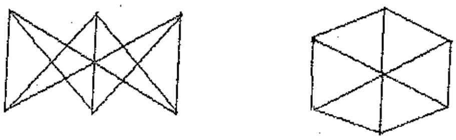


Fig.1(b)

(05 Marks)

c. Define i) Complete graph ii) Induced subgraph iii) Euler circuit. Give one example for each. **(05 Marks)**

d. If G is an undirected graph with n vertices and e edges, let $\delta = \min_{v\in\Upsilon}\{\deg(v)\}$ and let $\Delta = \max_{v\in\Upsilon}\{\deg(v)\}$, then prove that $\delta \le 2(e/n) \le \Delta$. **(05 Marks)**

2 a. Define Hamilton cycle. How many edge-disjoint Hamilton cycles exist in the complete graph with seven vertices? Also, draw the graph to show these Hamilton cycles. **(05 Marks)**

b. Define : i) Planar graph ii) Bipartite graph iii) Complete bipartite graph. Give one example for each. **(05 Marks)**

c. If G is a connected simple planar graph with n ($\ge$3) vertices, e($>$2) edges and r regions, then prove that i) $3r \le 2e$ ii) $e \le 3n - 6$. **(05 Marks)**

d. Define chromatic number. Find the chromatic polynomial for the cycle of length 4 as shown in Fig.2(d) below. Hence find the chromatic number. **(05 Marks)**

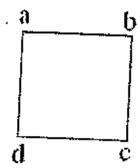


Fig.2(d)

3 a. Define a tree.
   i) Prove that a tree with two or more vertices contains at least two pendant vertices.
   ii) Suppose that a tree T has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4. Find the number of pendant vertices in T. **(06 Marks)**

b. Define : (i) Binary rooted tree (ii) Balanced tree.
Draw all the spanning trees of the graph show in Fig.3(b) below. **(07 Marks)**

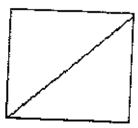


Fig.3(b)

c. Define prefix code. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. **(07 Marks)**

4 a. Explain Dijkstra's algorithm. (06 Marks)

b. State Kruskal's algorithm. Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown in Fig.4(b) below. (07 Marks)

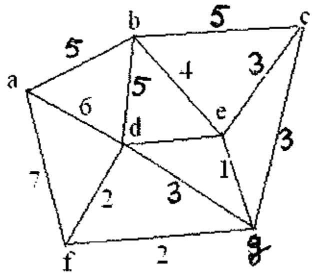


Fig.4(b)

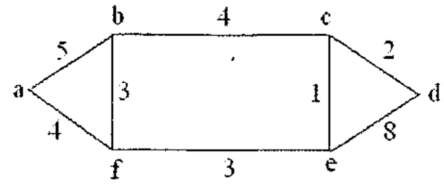


Fig.4(c)

c. Define a cut-set. For the network shown in Fig.4(c), find the capacities of all the cutsets between the vertices a and d, and hence determine the maximum flow between a and b. (07 Marks)

PART – B

5 a. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000? (05 Marks)

b. In how many ways can 10 identical dimes be distributed among five children if (i) there are no restrictions (ii) each child gets at least one dime (iii) the oldest child gets at least two dimes. (05 Marks)

c. Determine coefficient of xyz^2 in the expansion of $(2x - y - z)^4$. (05 Marks)

d. Define Catalan number. Using the moves $R : (x, y) \to (x+1, y)$ and $v : (x, y) \to (x, y+1)$, find in how many ways can one go

i) From (2,1) to (7,6) and not rise above the line $y = x - 1$.

ii) From (3,3) to (10,15) and not rise above the line $y = x + 5$. (05 Marks)

6 a. How many integers between 1 and 300 (inclusive) are

i) divisible by at least one of 5, 6, 8?

ii) divisible by none of 5, 6, 8? (06 Marks)

b. Define derangement. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that at least one letter gets to the right person. (07 Marks)

c. Find the rook polynomial for the 3×3 board using the expansion formula. (07 Marks)

7 a. i) Find a generating function for the sequence $1^2, 2^2, 3^2, \ldots\ldots$

ii) Find the coefficient of x^n in the expansion of $(x^2 + x^3 + x^4 + \ldots\ldots)^4$ (06 Marks)

b. Use generating function to determine in how many ways can two dozen identical robots be assigned to four assembly lines with i) at least 3 robots assigned to each line ii) at least 3 but not more than 8 robots assigned to each line. (07 Marks)

c. Using exponential generating function, find the number of ways in which 4 of the letters in ENGINE be arranged. (07 Marks)

8 a. Find and solve a recurrence relation for the number of binary sequences of length $n \geq 1$ that have no consecutive 0's. (06 Marks)

b. Solve the recurrence relation

$$a_{n+2} + 3a_{n+1} + 2a_n = 3^n \text{ for } n \geq 0; \text{ given } a_0 = 0,\ a_1 = 1.$$ (07 Marks)

c. Find a generating function for the recurrence relation

$$a_{n+2} - 2a_{n+1} + a_n = 2^n \text{ for } n \geq 0; \text{ given } a_0 = 1,\ a_1 = 2.$$

Hence solve it. (07 Marks)

* * * * *

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

USN | | | | | | | | | | | **06CS42**

Fourth Semester B.E. Degree Examination, Dec.09/Jan.10

Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Determine $|V|$ for the following graphs.

i) G has nine edges and all vertices have degree 3.

ii) G is regular with 15 edges. **(06 Marks)**

b. Define isomorphism of graphs. Show that no two of the following three graphs as shown in Fig.1(b) are isomorphic. **(07 Marks)**

Fig.1(b).

c. Define Euler circuit. Discuss Konigsberg bridge problem. **(07 Marks)**

2 a. Define: i) Bipartite graph ; ii) Hamilton cycle and iii) Planar graph. Give one example for each. **(06 Marks)**

b. If G = (V, E) is a loop-free connected planar graph with $|V| = n$, $|E| = e > 2$, and r regions, then prove that : i) $e \geq \frac{3r}{2}$; ii) $e \leq 3n - 6$. Further, if G is triangle free, then iii) $e \leq 2n - 4$. **(07 Marks)**

c. Define chromatic number. Find the chromatic polynomial for the cycle of length 4. Hence find its chromatic number. **(07 Marks)**

3 a. Define a tree. Prove that in every tree $T = (V, E)$, $|V| = |E| + 1$. **(06 Marks)**

b. Define: i) Rooted tree ; ii) Complete binary tree and iii) Spanning tree. Give an example for each. **(07 Marks)**

c. Obtain an optimal prefix code for the message ROAD IS GOOD using labelled binary tree. Indicate the code. **(07 Marks)**

4 a. State Max-flow and Min-cut theorem. For the network as shown in Fi.4(a), determine the maximum flow between the vertices A and D by identifying the cut-set of minimum capacity. **(06 Marks)**

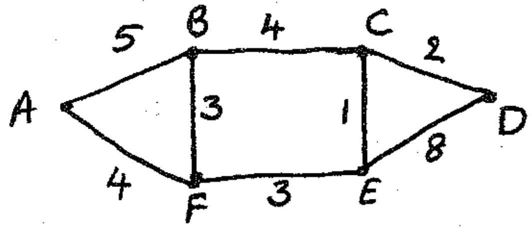


Fig.4(a).

b. State Kruskals algorithm. Apply Kruskal's algorithm to find a minimal spanning tree for the weighted graph as shown in Fig.4(b). **(08 Marks)**

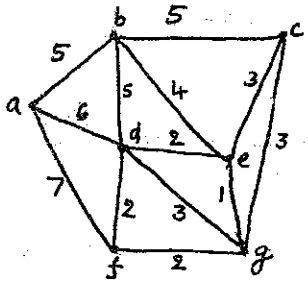


Fig.4(b).

c. Explain the steps in Dijikstra's shortest path algorithm. **(06 Marks)**

PART – B

5 a. i) How many distinct four digit integers can one make from the digits 1, 3, 3, 7, 7 and 8?
ii) Find the number of arrangements of the letters in TALLAHASSEE which have no adjacent A's **(06 Marks)**

b. In how many ways can 10 identical dime be distributed among five children if,
i) There are no restrictions
ii) Each child gets at least one dime
iii) The oldest child gets at least two dimes. **(07 Marks)**

c. Define Catalan number. In how many ways can one arrange three 1's and three – 1's so that all six partial sums (starting with the first summand) are nonnegative? List all the arrangements. **(07 Marks)**

6 a. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. Define derangement. Find the number of derangements of 1, 2, 3, 4. List all the derangements. **(07 Marks)**

c. A girl student has sarees of 5 different colours: blue, green, red, white and yellow. On Mondays, she does not wear green; on Tuesdays, blue or red; on Wednesday, blue or green; on Thursdays red or yellow; on Fridays, red. In how many ways can she dress without repeating a colour during a week (from Monday to Friday)? **(07 Marks)**

7 a. Find a generating function for each of the following sequences:
i) $1^2, 2^2, 3^2, 4^2, \ldots\ldots\ldots$
ii) $8, 26, 54, 92, \ldots\ldots\ldots\ldots$ **(06 Marks)**

b. Using the generating function, find the number of ways of forming a committee of 9 students drawn from 3 different classes so that students form the same class do not have an absolute majority in the committee. **(07 Marks)**

c. If a leading digit 0 is permitted, using exponential generating function, find the number of r – digit binary sequences that can be formed using an even number of 0's and an odd number of 1's. **(07 Marks)**

8 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

b. Solve the following recurrence relations:
i) $a_n - 3\, a_{n-1} = 5\,(3^n),\ n \geq 1,\ a_0 = 2.$
ii) $a_{n+2} + 4\, a_{n+1} + 4\, a_n = 7,\ n \geq 0,\ a_0 = 1,\ a_1 = 2.$ **(08 Marks)**

c. Find the generating function for the recurrence relation:
$a_{n+2} - 2\, a_{n+1} + a_n = 2^n,\ n \geq 0$ with $a_0 = 1,\ a_1 = 2$. Hence solve it. **(06 Marks)**

* * * * *