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06CS34

Third Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. Using Venn diagram, prove that, for any sets A, B and C $\overline{\{\overline{(A \cup B) \cap C}\} \cup \overline{B}} = B \cap C$. **(05 Marks)**

b. State and prove De Morgan's Laws of set theory. **(04 Marks)**

c. In a survey of 260 college students, the following data were obtained:
64 had taken a mathematics course, 94 had taken a computer science course, 58 had taken a business course, 28 had taken both a mathematics and a business course, 26 had taken both a mathematics and a computer science course, 22 had taken both a computer science and a business course, and 14 had taken all three types of courses.
i) How many of these students had taken none of the three courses?
ii) How many had taken only a computer science courses? **(06 Marks)**

d. Prove, by mathematical induction $1.3 + 2.4 + 3.5 + ...n(n+2) = \frac{n(n+1)(2n+7)}{6}$. **(05 Marks)**

2 a. If statement q has the truth value 1, determine all truth value assignments for the primitive statements p, r and s for which the truth value of the statement:
$(q \to [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \to (\neg r \wedge q)]$ is 1. **(04 Marks)**

b. Define tautology. Show that $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology by constructing truth table. **(05 Marks)**

c. Simplify the compound statement $\neg[\neg((p \vee q) \wedge r) \vee \neg q]$ using laws of logic. Mention the reasons. **(05 Marks)**

d. Write the following argument in symbolic form and then establish its validity:
If Rochelle gets the supervisor's position and works hard, then she'll get a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore, either Rochelle did not get the supervisor's position or she did not work hard. **(06 Marks)**

3 a. Define an open statement. Write the negation of the statement: If k, m, n are any integers where $k - m$ and $m - n$ are odd then $k - n$ is even. **(07 Marks)**

b. For the universe of all integers, define the following open statements:
p(x): x>0, q(x): x is even, r(x): x is a perfect square, s(x): x is divisible by 4 and t(x): x is divisible by 5.
Write the following statements in symbolic form and determine whether each of the statements is true or false. For each false statement, provide a counter example.
i) Atleast one integer is even ii) There exists a positive integer that is even iii) If x is even, then x is not divisible by 5 iv) If x is even and x is a perfect square, then x is divisible by 4. **(07 Marks)**

c. Give: i) A direct proof, ii) An indirect proof and iii) Proof by contradiction, for the following statement, "If n is an odd integer, then n+9 is an integer". **(06 Marks)**

4 a. Let A = {1, 2, 3, 4, 6} and 'R' be a relation on 'A' defined by aRb if and only if 'a' is multiple of 'b': i) Write down R as a set of ordered pairs ii) Represent R as a matrix iii) Draw the digraph of R. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 5} × {1, 2, 3, 4, 5}, and define R on A by
$(x_1, y_1)R(x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.
i) Verify that R is an equivalence relation on A.
ii) Determine the equivalence classes [(1, 3)], [(2, 4)] and [(1, 1)]. **(08 Marks)**

c. Define a poset. Consider the Hasse diagram of a poset (A, R) given below in fig.Q4(c). If B = {c, d, e}, find (if they exist).
i) The least upper bound of B ii) The greatest lower bound of B. **(06 Marks)**

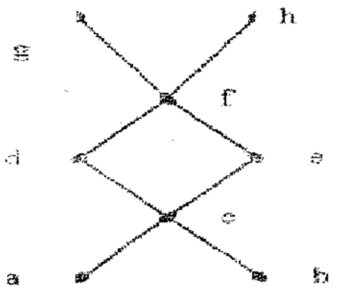

Fig.Q4(c)

5 a. Let S = {-1, 0, 2, 4, 7}. Find f(S) if i) $f(x) = 1$, ii) $f(x) = 2x+1$, iii) $f(x) = \left\lceil \frac{x}{5} \right\rceil$,
iv) $f(x) = \left\lfloor \frac{(x^2+1)}{3} \right\rfloor$. **(06 Marks)**

b. Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6}
i) How many functions are there from A to B? How many of these are one-to-one? How many are onto?
ii) How many functions are there from B to A? How many of these are onto? How many are one-to-one? **(06 Marks)**

c. Let A = B = R. Determine $\pi_A(D)$ and $\pi_B(D)$ for each of the following sets $D \subseteq A \times B$.
i) $D = \{(x, y)/x = y^2, 0 \le y \le 2\}$ ii) $D = \{(x, y)/y = \text{Sin}x, 0 \le x \le \pi\}$. **(08 Marks)**

6 a. Let f : R → R be defined by, $f(x) = \begin{cases} 3x-5, & x > 0 \\ -3x+1, & x \le 0 \end{cases}$. Find $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(3)$ and $f^{-1}([-5, 5])$. **(06 Marks)**

b. Prove that a function f : A → B is invertible if and only if it is one-to-one and onto. **(08 Marks)**

c. Prove that if 151 integers are selected from {1, 2, 3, …., 300}, then the selection must include two integers x, y where $x \mid y$ or $y \mid x$. **(06 Marks)**

7 a. Define the binary operation ∘ on Z by $x \circ y = x + y + 1$. Verify that (Z, ∘) is an Abelian group. **(05 Marks)**

b. Let (G, •) and (H, *) be two groups with respective identities e_G, e_H. If f : G → H is a homomorphism, then prove that i) $f(e_G) = e_H$ ii) $f(a^{-1}) = [f(a)]^{-1}$ for all a ∈ G
iii) f(S) is a subgroup of H for each subgroup S of G. **(08 Marks)**

c. Define cyclic group. Prove that every subgroup of a cyclic group is cyclic. **(07 Marks)**

8 a. Define group code. Let $E : Z_2^m \to Z_2^n$, m < n be the encoding function given by a generator matrix G or the associated parity – check matrix H. Prove that $C = E(Z_2^m)$ is a group code. **(10 Marks)**

b. Define a ring and an integral domain. Let R be a commutative ring with unity. Prove that R is an integral domain if and only if for all a, b, c ∈ R, where a ≠ z, (additive identity) $ab = ac \Rightarrow b = c$. **(10 Marks)**

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Third Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Discrete mathematical structures

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. Define the difference and symmetric difference, Δ of sets A and B.
If U = {1, 2, 3, - - - -, 9, 10}, A = {1, 2, 3, 4, 5}, B = {3, 4, 5, 6, 7} find AΔB, A–B and B–A. **(05 Marks)**

b. Dolly rolls a die three times. What is the probability that the result of her second roll is greater then that of her first roll and the result of her third roll is greater than the second? **(05 Marks)**

c. Using the principle of mathematical induction prove that

$$1.3 + 2.4 + 3.5 + - - - - + n(n+2) = \frac{n(n+1)(2n+7)}{6}.$$ **(05 Marks)**

d. Give the recursive definition of Fibonacci numbers and use it to prove that

$$\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}, \forall n \in Z^+ .$$ **(05 Marks)**

2 a. What is an implication and biconditional? Give their truth tables.
Determine the truth value of each of the following : **(07 Marks)**
i) If 3 + 4 = 12, then 3 + 2 = 6
ii) If 4 + 4 = 8, then 5 + 4 = 10
iii) If Dr. Radhakrishnan was the first president of India, then 3 + 4 = 7. **(07 Marks)**

b. Use truth tables to verify
$[P \rightarrow (q \cap r)] \Leftrightarrow [(p \rightarrow q) \cap (p \rightarrow r)]$

c. Express each of the following statements in symbols, negate them and write in smooth English.
i) Vimala will get a good education if she puts her studies before her interest in cheer leading.
ii) Nirma is doing her home work and kamala is practicing her music lessons. **(06 Marks)**

3 a. Define an argument and valid argument. Give examples to each.
Determine all truth value assignment for the primitive statements for the valid argument.
$p \wedge (q \wedge r) \rightarrow (s \veebar t)$ **(07 Marks)**

b. Give reasons for each step needed to show that the following argument is valid, using rules of inference :
$[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$ **(07 Marks)**

c. Let p(x, y), q(x, y) and r(x, y) represent open statements, with replacements for the variables x, y chosen from some prescribed universe. Write the negation mentioning the rules used.
$\forall x \exists y [(p(x, y) \wedge q(x, y)) \rightarrow r(x, y)]$ **(06 Marks)**

4 a. Define an Equivalence relation.
If A = {1, 2, 3, 4} give an example of a relation R on set A which is
i) Reflexive and symmetric, but not transitive. ii)Reflexive and transitive, but not transitive. iii) Symmetric and transitive, but not reflexive. **(07 Marks)**

b. For A = {1, 2, 3, 4}, let R_1 and R_2 be the relations on A defined by R_1 = {(1, 2), (1, 3), (2, 4), (4, 4)} and R_2 = {(1, 1), (1, 2), (1, 3), (2, 3), (2, 4)}. Find $R_1.R_2$, $R_2.R_1$, $R^2{}_1$, $R^2{}_2$, $R^3{}_1$. **(07 Marks)**

c. For A = {a, b, c, d, e, f}each graph and digraph in the following figures 4(c)(i) and 4(c)(ii) represents a relations R_1 and R_2 on A. Determine the relations $R_1, R_2 \subseteq A \times A$ in each case and its associated relation matrixes $M(R_1)$ and $M(R_2)$.

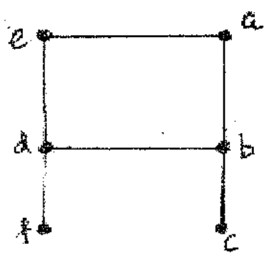

Fig. 4(i)

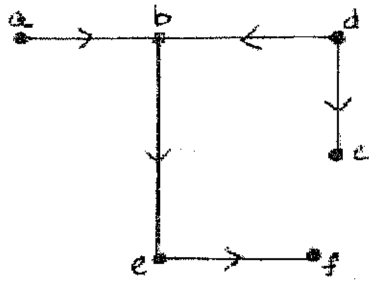

Fig. 4(ii) **(06 Marks)**

5 a. Define a partition and an equivalence class a set A.
If R is the relation on Z given by xRy if 4/x–y, find the equivalence classes [0], [1], [2], [3]. **(07 Marks)**

b. What is partial order and a partially ordered set? Draw the Horse diagram for the poset $(P(A), \subseteq)$ where A = {1, 2, 3, 4} and P(A) is the power set of A. **(07 Marks)**

c. What is a lattice?
Let R be a relation "exactly divides" on the set A = {2, 3, 5, 6, 7, 11, 12, 35, 385}. Is the poset (A, R) a lattice? Give reasons. **(06 Marks)**

6 a. Define : i) Bijective function. ii) Composite function.
Give an example in each case. **(07 Marks)**

b. Let $f : A \rightarrow B$ and $g : B \rightarrow C$. prove that
i) If f, g are one–to–one, then g of is one–to–one.
ii) If f, g are onto, the g of is onto. **(07 Marks)**

c. Let A = {1, 2, 3, 4, 5, 6}. Compute the product of the permutations (4, 1, ,3 5).(5, 6, 3) and (5, 6, 3).(4, 1, 3, 5) where (4, 1, 3, 5) and (5, 6, 3) are permutations on set A. **(06 Marks)**

7 a. Define a group and an abelian group. If the binary operation. on Z is given by x.y =x +y+1, verify that (Z, .) is an abelian group. **(07 Marks)**

b. State and prove Lagrange's theorem in groups. **(07 Marks)**

c. What is a group homomorphism and an isomorphism? Give example to each. **(06 Marks)**

8 a. Define : i) Hamming metric ii) The sphere of radius k centered at x Give an example in each case. **(07 Marks)**

b. What is a generator matrix? The encoding function $E : Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

i) Determine all code words, what is the error detection capability.
ii) Find the associated parity check matrix H. **(07 Marks)**

c. Define a ring and gives an example. **(06 Marks)**

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CSI32

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Third Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. Define the following with examples:
i) Symmetric difference ii) Complement of sets iii) Power set. **(06 Marks)**

b. Show that for any two sets A, B and C
i) $A-(A\cap B)=A-B$ ii) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$. **(08 Marks)**

c. Define the characteristic function of a set and prove that:
$f_{A\oplus B}=f_A+f_B-2f_Af_B$. **(06 Marks)**

2 a. Given $R=\{(x,y):x+3y=12\}$
i) Write R as a set of ordered pairs ii) Find domain, range, inverse of R iii) Find R.R. **(08 Marks)**

b. A = {a, b, c}, R = {(a, a) (a, b) (b, c) (c, c)}, find reflexive closure, symmetric closure and transitive closure. **(12 Marks)**

3 a. Show that, if any 11 numbers are chosen from the set {1, 2, ….20}, then one of them will be a multiple of another. **(06 Marks)**

b. In how many ways can six men and six women be seated in a row, if
i) Any person may sit next to any other ii) Men and women must occupy alternate seats. **(06 Marks)**

c. Let A = {1, 2, 3, 4, 5} and R be a relation defined by aRb if and only if a < b compute R^2, R^3, R^{-1}. **(08 Marks)**

4 a. If a set A has 'n' elements prove that its power set A has 2^n elements. **(08 Marks)**

b. Prove that poset had atmost one greater element and atmost one least element. **(06 Marks)**

c. Prove by induction method $\frac{1}{1.2}+\frac{1}{2.3}+\ldots\ldots\frac{1}{n(n+1)}=\frac{n}{n+1}$. **(06 Marks)**

5 a. Let R and S be equivalence relations on A. Then show that $R\cap S$ is an equivalence relation on A. **(06 Marks)**

b. Let A = Z^+, the set of positive integers, and let $R=\{(a,b)\in A\times A;\ a \text{ divides } b\}$. Is R reflective symmetric, asymmetric or antisymmetric? **(08 Marks)**

c. Write the Warshall algorithm for finding the transitive closure of relation. **(06 Marks)**

6 a. Find explicit formula for the Fibonacci sequence by formulating. **(08 Marks)**

b. Draw the Hasse diagram for factors of 36. **(08 Marks)**

c. Define the following function with example:
i) One to one function ii) On to function. **(04 Marks)**

7 a. Find the disfunctive normal form for $P\wedge(P\rightarrow q)$ and find conjunctive normal form for $P\wedge(P\rightarrow q)$. **(08 Marks)**

b. Define the external elements of poset. Give one example to each. **(06 Marks)**

c. Show that (A, ≤) is a poset and for all a, b in A, GLB(a, b) = a * b. **(06 Marks)**

8 Write short notes on:
a. Tautology b. Warshall's algorithm c. Quotient of a group d. Digraphs. **(20 Marks)**

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Third Semester B.E. Degree Examination, June / July 08
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions.*

1 a. Let p, q, r denote primitive statements. Use truth table to verify the following logical equivalences :
i) $p \to (q \wedge r) \Leftrightarrow (p \to q) \wedge (p \to r)$
ii) $[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$
iii) $[p \to (q \vee r)] \Leftrightarrow [-r \to r(p \to q)]$. **(06 Marks)**

b. What is tautology? Verify if the following statements are tautology :
i) $p \to [\to (p \wedge q)]$
ii) $(p \vee q) \to [q \to q]$
iii) $(p \vee q) \to [q \to (p \wedge q)]$. **(06 Marks)**

c. State identify and Absorption laws. Prove them using truth tables. **(08 Marks)**

2 a. Write the rule of inference, related logical implication and state proper examples for law of syllogism and Modus Tollens. **(08 Marks)**

b. For given open statements $p(x) : (x) > 3$ $q(x) : x > 3$, the universe consists of all real numbers. Express in statement and symbolic form, the converse, inverse and contra positive of the statement $\forall x[p(x) \to q(x)]$. State an example if the truth value is false. **(06 Marks)**

c. Prove the following arguments :
i) If m is an even integer, then prove m + 7 is odd.
ii) If k and ℓ are odd integers, prove $k + \ell$ is even. **(06 Marks)**

3 a. State and prove De Morgan's laws of set theory. **(06 Marks)**

b. In a survey of 120 passengers, an airline found that 48 enjoyed wine with their meals, 78 enjoyed mix drinks and 66 enjoyed icedtea. In addition 36 enjoyed any given pairs of these beverages and 24 enjoyed them all. If two passengers are selected at random from the survey sample of 120, what is the probability that –
i) Both want only icedtea
ii) Both enjoy exactly 2 of the 3 beverages offered.
Solve using Venn diagram and probability. **(08 Marks)**

c. At Wimbeldon Tennis Championship, women play at most 3 sets in a match. The winner is the first to win 2 sets. Draw the tree diagram and sample space, and find number of ways in which the match can be won. **(06 Marks)**

4 a. State and briefly explain mathematical induction. **(04 Marks)**

b. Prove by mathematical induction that
$$1^2 + 2^2 + 3^2 + - - - - + n^2 = \frac{n(n+1)(2n+1)}{6}.$$ **(08 Marks)**

c. For given set $A = \{1,2,3,4\}, B = \{w, x, y, z\}, C = \{5,6,7\}$,
$R_1 = \{(1,x)(2,x)(3,y)(3,z)\}$
$R_2 = \{(w,5)(x,6)\}$
Compute $M(R_1)$, $M(R_2)$ and hence find $M(R_1 \cdot R_2)$. **(10 Marks)**

5 a. For a binary function $f : A \times A \to B$, state the commutative and associative property. For closed binary operation. $F : Z \times Z \to Z$ defined by $f(a, b) = a + b - 3ab$, prove that f commutative as well as associative for all $a, b, c \in Z$. **(08 Mark**

b. What is a composite function? for $A = \{1, 2, 3, 4\}$ $B = \{a, b, c\}$, $C = \{w, x, y, z\}$ wi $f : A \to B$ and $g : B \to C$ given by $f = \{(1, a) (2, a) (3, b) (4, c)\}$, $g = \{(a, x) (b, y) (c, z)\}$ Find gof. **(04 Mark**

c. What are reflexive, symmetric, transitive and antisymmetric relations? State an examp for each. **(08 Mark**

6 a. Define Cartesian product

For $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 5\}$, $C = \{3, 4, 7\}$.

Find – i) $A \times B$ ii) B^2 iii) $(A \times C) \cup (B \times C)$ iv) $A \cup (B \times C)$. **(06 Mark**

b. If a coin is tossed 3 times, what is the probability of getting 2 heads and 1 tail? Show th sample space and tree diagram. **(10 Mark**

c. What is function? How is it different from a relation? Explain the different types c functions with an example for each. **(06 Mark**

7 a. Let G be the set of all non zero real numbers. For $a * b = \frac{ab}{2}$, show that $(G,*)$ is an Ablia group. **(06 Marks**

b. For a group G with a and b as elements of G, show that

i) $(a^{-1})^{-1} = a$, ii) $(ab)^{-1} = b^{-1} a^{-1}$. **(06 Marks**

c. Consider the set Z with binary operation $\oplus$ and $\odot$ which are defined by $x \oplus y = x + y -$ $x \odot y = x + y - xy$ show that $(Z, \oplus, \odot)$ is a ring. **(08 Marks**

8 a. Write a note on binary operations and their properties. **(10 Marks**

b. Given 'R' a non empty set with two closed operations denoted by + and ·, explain wher (R, +, ·) can be a ring.

c. Define the following :

i) Semi group and Abelian group.

ii) Left coset and right coset

iii) Homomorphism and isomorphism of a group

iv) Congruence modulo n with an example. **(10 Marks)**

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CSI32

Third Semester B.E. Degree Examination, June/July 08

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. If A and B are any two finite sets, then prove that $A-(A-B)=A\cap B$. **(06 Marks)**

b. Define and give the examples for each : i) Power set ii) Finite set iii) Null set **(06 Marks)**

c. How many different eleven person cricket team can be formed each containing 5 female members from an available set of 20 females and 6 male members from an available set of 30 males. **(04 Marks)**

d. Define extended Pigeon hole principle. Show that if any 30 students are selected, then you may choose a subset of 5 so that all 5 were born on the same day of the week. **(04 Marks)**

2 a. Given $R=\{(x,y): x+3y=12\}$

i) Write R as a set of ordered pairs ii) Find domain, range, inverse of R iii) Find R.R. **(08 Marks)**

b. A = {a, b, c}, R = {(a, a) (a, b) (b, c) (c, c)}, find reflexive closure, symmetric closure and transitive closure. **(12 Marks)**

3 a. Define partition set, anti-symmetric relation, equivalence relation, transitive closure. **(04 Marks)**

b. Let A = {a, b, c, d, e} and R = {(a, a) (a, b) (b, c) (c, e) (c, d) (d e)}. Compute R^2 and R^∞. **(06 Marks)**

c. Let A = {1, 2, 3, 4} and relation R = {(1, 2) (2, 1) (2, 3) (3, 4) (4, 4)}. Find matrix of transitive closure of R by using Warshall's algorithm. **(10 Marks)**

4 a. If a set A has 'n' elements prove that its power set A has 2^n elements. **(08 Marks)**

b. Prove that poset had atmost one greater element and atmost one least element. **(06 Marks)**

c. Prove by induction method $\frac{1}{1.2}+\frac{1}{2.3}+\dots\dots\frac{1}{n(n+1)}=\frac{n}{n+1}$. **(06 Marks)**

5 a. Prove that the congruence relation is an equivalence relation. **(06 Marks)**

b. Let A = {1, 2, 3, 4}, R = {(1, 2) (2, 3) (3, 4) (2, 1)}, find the transitive closure of R. **(08 Marks)**

c. Let R be an equivalence relation on a set S, then prove that the quotient S/R is a partition of S. **(06 Marks)**

6 a. Define the following function with example,

i) One to one function. ii) On to function. **(04 Marks)**

b. Let $f(x)=(x+2)$, $g(x)=(x-2)$, $h(x)=3x$ for $x\in R$ where R is the set of real numbers. Find $g\circ f$, $g\circ g$, $h\circ f$ and $f\circ h\circ g$. **(08 Marks)**

c. Let F_x be the set of all one – to – one and on to mapping from x onto x, where $x=\{1, 2, 3\}$ Find all the elements of F_x and find the inverse of each element. **(08 Marks)**

7 a. Find the disfunctive normal form for $P\wedge(P\to q)$ and find conjunctive normal form for $P\wedge(P\to q)$. **(08 Marks)**

b. Define the external elements of poset. Give one example to each. **(06 Marks)**

c. Show that (A, ≤) is a poset and for all a, b in A, GLB(a, b) = a * b. **(06 Marks)**

8 Write short note on:

a. Tautology.

b. Warshall's Algorithm.

c. Lattices.

d. Digraphs. **(20 Marks)**

USN | | | | | | | | | | | **06CS34**

Third Semester B.E. Degree Examination, June / July 08
Discrete Mathematical Structures

Time: 3 hrs.   Max. Marks:100

Note : ***Answer any FIVE full questions, choosing at least TWO from each part.***

PART - A

1 a. Define the following terms and give an example for each i) Set ii) Proper subset iii) Power set iv) Empty set v) Venn diagram. **(05 Marks)**

b. Using the laws of set theory, simplify each of the following

i) $A \cap (B-A)$ ii) $\overline{\overline{(A \cup B) \cap C} \cup \overline{B}}$ **(05 Marks)**

c. In a class of 30 students, 15 take arts, 8 take science, 6 take commerce, 3 take all the three courses. Show that 7 or more students take none of the course. **(05 Marks)**

d. A problem is given to four students A, B, C, D whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ respectively. Find the probability that the problem is solved. **(05 Marks)**

2 a. Let p, q be primitive statements for which the implication $p \rightarrow q$ is false. Determine the truth values for each of the following. i) $p \wedge q$ ii) $\neg p \vee q$ iii) $q \rightarrow p$ iv) $\neg q \rightarrow \neg p$. **(05 Marks)**

b. Verify that $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$, for primitive statements p, q, and r. **(05 Marks)**

c. Prove the following logical equivalence using the laws of logic : $(\neg p \vee \neg q) \wedge (F_0 \vee P) \wedge P$. **(05 Marks)**

d. Establish the validity of the following argument using the rules of Inference.
$[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$. **(05 Marks)**

3 a. What are the bound variables and free variables? Identify the same in each of the following expressions : i) $\forall_y \exists_z [\cos(x+y) = \sin(z-x)]$ ii) $\exists_x \exists_y [x^2 - y^2 = z]$. **(05 Marks)**

b. Let p(x) be the open statement "$x^2 = 2x$", where the universe comprises all integers. Determine whether each of the following statements is true or false. i) P(0) ii) P(1) iii) P(2) iv) P(-2) v) $\exists x\ P(x)$. **(05 Marks)**

c. Provide the steps and reasons to establish the validity of the argument :

$$\begin{array}{l} \forall x[p(x) \rightarrow (q(x) \wedge r(x))] \\ \underline{\forall x[p(x) \wedge s(x)]} \\ \therefore \forall x[r(x) \wedge s(x)] \end{array}$$

(05 Marks)

d. Give a direct proof for each of the following
i) For all integers K and *l*, if k, *l* are both even, then k+*l* is even.
ii) For all integers k and *l*, if k, *l* are even, then k.*l* is even. **(05 Marks)**

4 a. Prove by Mathematical Induction that for every positive integer n, n 1, 2^{n-1}. **(07 Marks)**

b. Apply backtracking technique to obtain an explicit formula for the sequence, defined by the recurrence relation $b_n = 2b_{n-1} + 1$ with initial condition $b_1 = 7$. **(06 Marks)**

c. Solve the linear recurrence relation $a_n = 4a_{n-1} + 5\ a_{n-2}$ with $a_1 = 2$, $a_2 = 6$. **(07 Marks)**

PART - B

5 a. Define the following terms and give an example for each i) Reflexive ii) Irrefle
iii) antisymmetric iv) Transitive v) partition set. (05 M

b. For $A = \{1, 2, 3, 4\}$ and $R = \{ (1, 2), (1, 3), (2, 4), (4,4)\}$. Compute i) R^2 ii) R^3 iii)
iv) M_R v) $(M_R)^t$. (05 M

c. If $A = A_1 \cup A_2 \cup A_3$, where $A_1 = \{1,2\}$, $A_2 = \{2,3,4\}$ and $A_3 = \{5\}$, define relation R
by xRy if x and y are in the same subset A_i, for $1 \leq i \leq 3$. Is R an equivalence relation? (05 M

d. Draw the diagraph and Hasse diagram representing the positive divisors of 36. (05 M

6 a. For each of the following function, determine whether it is one-to-one and determin range.
i) $f : Z \to Z$, $f(x) = 2x+1$ ii) $f : Q \to Q$, $f(x) = 2x+1$ iii) $f : Z \to Z$, $f(x) = x$
iv) $f : R \to R$, $f(x) = e^x$. v) $f: [0, \pi] \to R$, $f(x) = \sin x$. (05 M

b. State the pigeonhole principle. Let ABC be an equilateral triangle with AB = 1. Show if we select five points in the interior of this triangle, there must be atleast two wl distance apart is less than ½. (05 M

c. Define function composition and let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$ and $C = \{w, x, y$ with $f : A \to B$ anf $g : B \to C$, given by $f = \{(1, a), (2, a), (3, b), (4, c)$ and $g = \{(a$ $(b, y), (c, z)\}$. For each of the element of A find gof. (05 M

d. Let $f, g : z^+ \to z^+$ where for all $x \in z^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x-1\}$, the maxin of 1 and x-1. i) What is the range of f? ii) Is f an onto function? iii) Is the func f one – to – one. iv) What is the range of g? v) Is g an onto function. (05 M

7 a. Define i) Group ii) Subgroup iii) homomorphism iv) Cyclic group v) Coset. (05 M

b. Prove that if G is a finite group of order n with H a subgroup of order m, then m divide (05 M

c. Let $G = S_4$, the symmetric group on four symbols and let H be the subset of G where

$$H = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \right\}.$$

Construct a table to show that H is an abelian group of G. (05 M

d. If G is a group, prove that for all a, b, $\in G$, i) $(a^{-1})^{-1} = a$ ii) $(ab)^{-1} = b^{-1}a^{-1}$. (05 M

8 a. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(R)$, prove that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a unit of this ring if and only if $ad - bc \neq 0$. (05 M

b. Let (R, +, .) be a commutative ring and let z denote the zero element of R. for a fi element $a \in R$, define $N(a) = \{r \in R \mid ra = Z\}$. Prove that N(a) is an ideal of R. (05 M

c. Construct a decoding table (with syndromes) for the code given by the generator

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

(05 M

d. Prove that for all $n \in N$, $10^n \equiv (-1)^n \pmod{11}$. (05 M

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CS34

USN

NEW SCHEME

Third Semester B.E. Degree Examination, July 2007
CS / IS
Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note : *Answer any FIVE full questions.*

1 a. For any three sets A, B, C prove that
$$(A - B) - C = A - (B \cup C) = (A - C) - (B - C)$$
(07 Marks)

b. A professor has two dozen introductory textbooks on computer science and is concerned about their coverage of the topics (A) compiler, (b) data structures and (C) operating systems. The following data are the numbers of books that contain material on these topics :
$|A| = 8,\ |B| = 13,\ |C| = 13,\ |A \cap B| = 5,\ |A \cap C| = 3,\ |B \cap C| = 6,\ |A \cap B \cap C| = 2$
i) How many of the textbooks include material on exactly one of these topics?
ii) How many do not deal with any of the topics?
iii) How many have no material on compilers?
(07 Marks)

c. For all $n \in Z^+$ show that if $n \geq 24$, then n can be written as a sum of $5's$ and or $7's$.
(06 Marks)

2 a. Define Tautology. Prove that, for any propositions p, q, r, the compound proposition $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology.
(05 Marks)

b. Prove the following logical equivalences without using truth tables :
i) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$
ii) $[(\neg p \vee \neg q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$
(05 Marks)

c. For any statements p, q prove that
i) $\neg(p \downarrow q) \Leftrightarrow (\neg p \uparrow \neg q)$
ii) $\neg(p \uparrow q) \Leftrightarrow (\neg p \downarrow \neg q)$
(05 Marks)

d. Test the validity of the following argument :
I will become famous or I will not become a musician
I will become a musician. Therefore, I will become famous.
(05 Marks)

3 a. Write down the following propositions in symbolic form and find its negation.
i) If all triangles are right angled, then no triangle is equiangular.
ii) For all integers n, if n is not divisible by 2, then n is odd.
(07 Marks)

b. Prove that the following argument is valid :
$$\forall x[p(x) \to q(x)]$$
$$\forall x[q(x) \to r(x)]$$
$$\therefore\ \forall x[p(x) \to r(x)]$$
where p(x), q(x) and r(x) are open statements that are defined for a given universe.
(07 Marks)

c. Provide a proof by contradiction for the following :
For every integer n, if n^2 is odd, then n is odd.
(06 Marks)

Contd.... 2

CSI32

USN | | | | | | | | | |

OLD SCHEME

Third Semester B.E. Degree Examination, July 2007

Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. If A and B are any two finite sets, then prove that $A-(A-B)=A\cap B$. **(06 Marks)**

b. Define and give the examples for each
   i) Power set.
   ii) Finite set.
   iii) Null set. **(06 Marks)**

c. How many different eleven person cricket team can be formed each containing 5 female members from an available set of 20 females and 6 male members from an available set of 30 males. **(04 Marks)**

d. Define extended Pigeon hole principle. Show that if any 30 students are selected, then you may choose a subset of 5 so that all 5 were born on the same day of the week. **(04 Marks)**

2 a. Prove by mathematical induction

$$1^3+2^3+3^3+\ldots\ldots+n^3=\frac{n^2(n+1)^2}{4}$$ **(07 Marks)**

b. Define
   i) Tautology
   ii) Contradiction
   iii) Contingence
   Give an examples for each. **(06 Marks)**

c. Find the negation of the following statement
   "All odd numbers are not prime numbers and some prime numbers are even". **(07 Marks)**

3 a. Define partition set, anti-symmetric relation, equivalence relation, transitive closure. **(04 Marks)**

b. Let A = {a, b, c, d, e} and R = {(a, a) (a, b) (b, c) (c, e) (c, d) (d e)}. Compute R^2 and R^∞. **(06 Marks)**

c. Let A = {1, 2, 3, 4} and relation R = {(1, 2) (2, 1) (2, 3) (3, 4) (4, 4)}. Find matrix of transitive closure of R by using Warshall's algorithm. **(10 Marks)**

4 a. Define invertible function. Let $f:R\rightarrow R$, is a function where R is the set of real numbers, defined by $f(x)=x^2$. Is f invertible? **(06 Marks)**

b. If $f:A\rightarrow B$ is a function and $|A|=|B|=n$, prove that
   i) If f is one to one, then f is onto.
   ii) If f is onto, then f is one to one. **(06 Marks)**

Contd....2

4 c. Define cyclic permutation. Write permutation

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$$ as a product of disjoint cycles. Is P an even or permutation?

(08 Ma

5 a. Prove that the congruence relation is an equivalence relation. (06 Ma

b. Let A = {1, 2, 3, 4}, R = {(1, 2) (2, 3) (3, 4) (2, 1)}, find the transitive closure of

(08 Ma

c. Let R be an equivalence relation on a set S, then prove that the quotient S/R i partition of S.

(06 Mar

6 a. Define Poset with an example. (04 Mar

b. Define maximal element, minimal element, greatest element and least element w examples.

(08 Mar

c. Let S = {a, b, c} and A = P(S). Write the powerset of S and draw the Hase diagra

(08 Mar

7 a. Define semigroup, commutative semigroup. If a semigroup (A, *) has an ident element, it is unique.

(06 Mark

b. Define isomorphism of two semigroups. Show that the semigroup (Z, +) and (E, - where E is the set of even integers, are isomorphic. (06 Mark

c. Define monoid. Let (S, *) and (T, *′) be monords with identities e and respectively. Let $f : S \to T$ be an isomorphism. Then prove that $f(e) = e'$.

(08 Mark

8 Write short notes on:

a. Watchall Algorithms.

b. Characteristic function.

c. Homomorphism.

d. Recurance relations.

(20 Mark

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CS34

USN

NEW SCHEME

Third Semester B.E. Degree Examination, Dec.06 / Jan.07
CS / IS
Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note: ***1. Answer any FIVE full questions.***

1 a. If $S, T \subseteq U$, prove that S and T are disjoint if and only if $S \cup T = S \,\Delta\, T$. **(04 Marks)**

b. Simplify the expression $\overline{\overline{(A \cup B \cap C)} \cup \overline{B}}$. **(04 Marks)**

c. A problem is given to four students A, B, C, D whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ respectively. Find the probability that the problem is solved. **(04 Marks)**

d. i) Prove by mathematical induction that for every positive integer n, 3 divides $n^3 - n$.
ii) Find the explicit formula for $c_n = c_{n-1} + n$, $c_1 = 5$. **(08 Marks)**

2 a. Let p and q be primitive statements for which $p \to q$ is false. Determine the truth values of the following :
i) $p \wedge q$ ii) $\sim p \vee q$ iii) $q \to p$ iv) $\sim q \to \sim p$ **(05 Marks)**

b. Prove the following logical equivalence without using truth table :
$(p \to q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim (q \vee p)$. **(05 Marks)**

c. Prove that for all integer k and l, if k and l are both odd then k + l is even and kl is odd. **(05 Marks)**

d. Find whether the following argument is valid :
No engineering student of first or second semester studies logic.
Anil is an engineering student who studies logic
Therefore Anil is not in second semester. **(05 Marks)**

3 a. Define partition set. List all partitions of P = {1, 2, 3} **(04 Marks)**

b. Let A = {a, b, c, d} and R = {(ab), (bb), (cb), (cd), (da), (ac)}
Compute i) R^2 ii) R^∞ iii) $M_R^{\,2}$ iv) $M_R^{\,6}$ **(04 Marks)**

c. Let A = {1, 2, 3, 4, 6, 12} on A. Define the relation R by aRb if and only if 'a' divides 'b'. Prove that R is a partial order on A. Draw the Hass diagram for this relation. **(04 Marks)**

d. Let A = {1, 2, 3, 4, 5} define a relation R on A × A by $(x_1, y_1)\, R\, (x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.
i) Verify that R is an equivalence on A × A.
ii) Determine the equivalence classes $[(1,3)], [(2,4)]$ and $[(1,1)]$.
iii) Determine the partition of A × A induced by R. **(08 Marks)**

4 a. In each of the following cases sets A and B and a function 'f' from A to B are given. Determine whether f is one to one or not or both or neither.
i) A = {1, 2, 3, 4} B = {1, 2, 3, 4} f = {(1 1), (2 3), (3 4), (4 2)}
ii) A = {a, b, c} B = {1, 2, 3, 4} f = {(a 1), (b 1), (c 3)}
iii) A = {1, 2, 3} B = {1, 2, 3, 4, 5} f = {(1 1), (2 3), (3 4)}
iv) A = {1, 2, 3} B = {1, 2, 3, 4, 5} f = {(1 1), (2 3), (3 3)} **(06 Marks)**

b. Let $f : A \to B$ and $g : B \to C$ be any two functions. Then the following are true.
i) If f and g are one-to-one so is $g \circ f$.
ii) If $(g \circ f)$ is one-to-one then f is one-to-one. **(04 Marks)**

Contd....2

4 c. Using characteristic function, prove that $(A \Delta B) \Delta C = A \Delta (B \Delta C)$. (06 Marks)

d. Find the minimal and maximal element of the poset. (04 Marks)

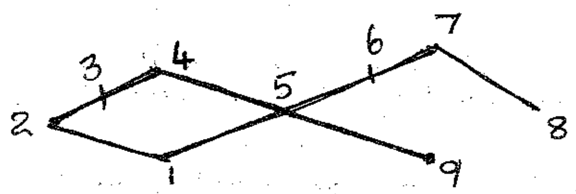

Fig. Q4 (d)

5 a. A function $f : A \rightarrow B$ is invertible if and only if it is one-to-one and onto. (06 Marks)

b. Let A = {1, 2, 3, 4, 5, 6, 7, 8} compute the products,
i) (2 5 8 6) ∘ (3 8 4) ii) (2 4) ∘ (3 5 7 1) ∘ (1 3 5 7) (06 Marks)

c. Let A = {1 2 3 4 5 6}. Determine the values of n such that $P^n = I_A$ for the following permutations $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{bmatrix}$. (04 Marks)

d. Let A = B = C = R and $f : A \rightarrow B$ and $g : B \rightarrow C$ be defined by f(a) = 3a − 1 and $g(b) = b^2 + 1$. Find i) (g ∘ f) (-2) ii) (f ∘ f)y. (04 Marks)

6 a. Prove that a group G in which every element is its own inverse is abelian. (05 Marks)

b. Define a sub group. Let G be a group and $G_1 = \{x \in G | xy = yx \text{ for } y \in G\}$. Prove that G_1 is a sub group of G. . (05 Marks)

c. Let H = {a, c, e} compute the left cosets of the elements of G = {a b c d e f} with respect to H. (05 Marks)

d. State and prove Lagrange's theorem. (05 Marks)

7 a. The generator matrix for an encoding function $E : Z_2^3 \rightarrow Z_2^6$ is given by $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$. Find the code words assigned to 110 to 010. Also obtain the associated parity check. (06 Marks)

b. Define a group Homomorphism and group Isomorphism. Let f be a homomorphism from a group G_1 and group G_2. Prove that,
i) If e_1 is the identity in G_1 and e_2 is the identity in G_2 then $f(e_1) = e_2$.
ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G$. (06 Marks)

c. Prove that the set Z with binary operation ⊕ and ⊙ defined by $x \oplus y = x + y - 1$, $x \odot y = x + y - xy$ is a commutative ring with unity. (08 Marks)

8 a. Define power set and give the power set of the following :
i) {a, {b}} ii) {1, φ, {φ}} (05 Marks)

b. Identify the following recurrence relation as linear homogeneous or not. If the relation is a linear homogeneous, give its degree.
i) $a_n + 2a_{n-1} + a_{n-2} = 0$ ii) $d_n = \sqrt{d_{n-1} + d_{n-2}}$
iii) $c_n = c_{n+1}^2 + c_{n-2}^2$ iv) $e_n - 5e_{n-1} + 6e_{n-3} - e_{n-5} = 0$
v) $b_n = b_{n-1} + 5$ (05 Marks)

c. Explain the relation 'R' on a set A is i) Reflexive ii) Symmetric iii) Transitive iv) Antisymmetric v) Irreflexive. (05 Marks)

d. Define i) maximal element ii) minimal element iii) greatest element iv) least element v) lattice. (05 Marks)

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CSI32

USN | | | | | | | | | | |

OLD SCHEME

Third Semester B.E. Degree Examination, Dec. 06 / Jan. 07
CSE / ISE
Discrete Mathematical Structure

Time: 3 hrs.] [Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define set, union of sets, disjoint sets. Give one example for each. **(06 Marks)**
b. Show that for any two sets A and B:
$\overline{A \cup B} = \overline{A} \cap \overline{B}$. **(07 Marks)**
c. In a survey of 260 college students, the following data were obtained.
64 had taken mathematics subject, 94 had taken physics subject, 58 had taken chemistry subject, 28 had taken both mathematics and chemistry, 26 had taken both mathematics and physics, 22 had taken both physics and chemistry. 14 had taken all three subjects.
i) How many students had taken exactly one subject?
ii) How many students had taken only mathematics?
iii) How many had taken none of the subjects? **(07 Marks)**

2 a. Let A = {0, 1}. Show that expressions i) $00^*(0 \vee 1)^*1$ and ii) $(01)^*(01 \vee 1^*)$ are regular expressions over A. **(06 Marks)**
b. Prove by mathematical induction:
$1 + 5 + 9 + \text{------} + (4n - 3) = n(2n - 1)$. **(07 Marks)**
c. Show that ${}^{n+1}C_r = {}^{n}C_{r-1} + {}^{n}C_r$ **(07 Marks)**

3 a. Define statement, disjunction, biconditional and tautology. **(06 Marks)**
b. Compute the truth table of the following:
$(p \to q) \leftrightarrow (\sim q \to \sim p)$. **(07 Marks)**
c. Show that if any seven numbers from 1 to 12 are chosen, then two of them will add up to 13. **(07 Marks)**

4 a. If $A \subseteq C$ and $B \subseteq D$ then prove that $A \times B \subseteq C \times D$. **(06 Marks)**
b. Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. **(07 Marks)**
c. Define domain and range of the relation. Let A = B = {1, 2, 3, 4, 5}; aRb if and only if a divides b. Write the relation and present it by matrix. **(07 Marks)**

5 a. Define equivalence relation. A is a set of all lines in a plane. $l_1 R l_2$ if and only if line l_1 is parallel to line l_2. Show that R is equivalence. **(06 Marks)**
b. Relation on the set A = {1, 2, 3, 4} is R = {(1,1), (1,3), (1,4), (2, 2), (2, 3), (2,4), (3,1), (3,4), (4,2)}. Construct a matrix and digraph of the relation. Write in-degrees and out-degrees of all vertices. **(07 Marks)**
c. Define complementary relation and inverse relation R and S are relations form A and B. Show that if $R \subset S$ then $R^{-1} \subset S^{-1}$. **(07 Marks)**

Contd.... 2

6 a. Define a function. If f: A → B and g: B → A defined by $f(a) = \frac{a+1}{2}$ an
g(b) = 2b – 1. Verify $f^{-1} = g$. **(06 Marks**

b. Let A = B = IR and f: A → B and g: B → C defined by f(x) = x + 1 an
$g(y) = 2y^3 - 1$. Find:
i) (fog)(2) ii) (gof)(2) iii) (gog)(2) iv) (fof)(2). **(07 Marks**

c. Define permutation. Let A = {1, 2, 3, 4, 5}. Compute (3, 2, 1, 5) ∘ (5, 4, 2) an
(5, 4, 2) ∘ (3, 2, 1, 5). **(07 Marks**

7 a. Define lattice. If U is any set and $(P(U), \subseteq)$ is a lattice, show that lub of A and B is $A \cup B$ and glb of A and B is $A \cap B$ where A, B belongs to $(P(U), \subseteq)$. **(06 Marks)**

b. If $(A, \leq)$ and $(B, \leq)$ are posets then show that $(A \times B, \leq)$ is a poset with partial orde
$\leq$ defined by $(a, b) \leq (a', b')$ if $a \leq a'$ in A and $b \leq b'$ in B. **(07 Marks)**

c. Show that in a Boolean algebra for any a and b
i) $(a \wedge b) \vee (a \wedge b') = a$
ii) $b \wedge (a \vee (a' \wedge (b \vee b'))) = b$. **(07 Marks)**

8 a. Define semigroup. If f is a homomorphism from a commutative semigroup $(S, *)$ onto a semigroup $(T, *')$, then show that $(T, *')$ is also commutative. **(06 Marks)**

b. Let G be a group and let a, b, $c \in G$, then show that:
i) $ab = ac \Rightarrow b = c$.
ii) $(ab)^{-1} = b^{-1} a^{-1}$. **(07 Marks)**

c. Define Abelian group. Let G be a group. Show that each element a in G has only one inverse in G. **(07 Marks)**

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CSI32

Reg. No. ☐☐☐☐☐☐☐☐☐☐

Third Semester B.E. Degree Examination, January/February 2006

Computer/Information Science and Engineering

(Old Scheme)

Discrete Mathematical Structures

Time: 3 hrs.) (Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. All questions carry equal marks.

1. (a) Define proposition, conjunction, disjunction, implication, tautology and give one example for each. **(10 Marks)**

(b) Prove that n^2 is odd if and only if n is odd, where n is an integer. **(5 Marks)**

(c) Show that the statement

$$a + ar + ar^2 + ... + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

is true by using mathematical induction. **(5 Marks)**

2. (a) How many different seven persons committee can be formed each containing 3 women from an available set of 20 women and 4 men from an available set of 30 men? **(7 Marks)**

(b) Show that if any five numbers from 1 to 8 are chosen, then two of them will add up to 9. **(6 Marks)**

(c) Use the technique of backtracking to find an explicit formula for the sequence defined by the recurrence relation $a_n = a_{n-1} + n$ and initial condition $a_1 = 4$ **(7 Marks)**

3. (a) Define partition set, antisymmetric relation, equivalence relation, transitive closure. **(4 Marks)**

(b) Let $A = \{a, b, c, d, e\}$ and $R = \{(a, a), (a, b), (b, c), (c, e), (c, d), (d, e)\}$. Compute R^2 and R^∞. **(6 Marks)**

(c) Let $A = \{1, 2, 3, 4\}$ and relation $R = \{(1, 2), (2, 1)(2, 3), (3, 4)(4, 4)\}$. Find matrix of transitive closure of R by using Warshall's algorithm. **(10 Marks)**

4. (a) Define invertible function. Let $f : R \to R$, where R be the set of real numbers, defined by $f(x) = x^2$. Is f invertible? **(6 Marks)**

(b) If $f : A \to B$ is a function and $|A| = |B| = n$, prove that (i) if f is one to one, then f is onto (ii) if f is onto, then f is one to one. **(6 Marks)**

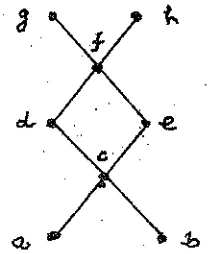
(c) Define cyclic permutation. Write permutation

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$$ as a product of disjoint cycles. Is P an even or odd permutation? **(8 Marks)**

5. (a) Define partial order relation and give an example. Let $S = \{a, b, c\}$ be a set and let p(s) be its power set. Draw the Hasse diagram of the poset p(s) under the inclusion relation $\subseteq$. **(6 Marks)**

Contd.... 2

(b) Define least upper bound, greatest lower bound of a subset. For the poset shown in the following Hasse diagram, find i) all upper bounds ii) all lower bounds iii) the least upper bound iv) the greatest lower bound of the set $B = \{c, d, e\}$ **(6 Marks)**



(c) Let L be a bounded distributive lattice. If a complement exists, it is unique. **(8 Marks)**

6. (a) Define isomorphism of two posets. Let A be the set z^+ of all positive integers and let $\leq$ be the usual partial order on A. Let A' be the set of positive even integers, and let $\leq'$ be the usual partial order on A'. Then show that the function $f : A \to A'$ given by f(a) = 2a is an isomorphic from $(A, \leq)$ to $(A', \leq')$ **(7 Marks)**

(b) Let L be a lattice then

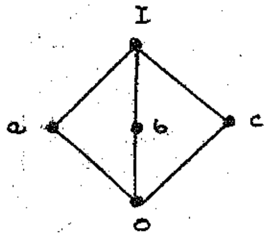
i) $a \vee (b \vee c) = (a \vee b) \vee c$

ii) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

iii) $a \vee b = b$ if and only if $a \leq b$

iv) $a \wedge b = a$ if and only if $a \leq b$ **(8 Marks)**

(c) Show that the lattice **(5 Marks)**



is nondistributive.

7. (a) Define semigroup, commutative semigroup. If a semigroup $(A, *)$ has an identity element, it is unique. **(6 Marks)**

(b) Define isomorphism of two semigroups. Show that the semigroup (Z, +) and (E, +), where E is the set of even integers, are isomorphic. **(6 Marks)**

(c) Define monoid. Let $(S, *)$ and $(T, *')$ be monoids with identities e and e', respectively. Let $f : S \to T$ be an isomorphism. Then $f(e) = e'$. **(8 Marks)**

8. (a) Define group and abelian group. Let G be a group. Each element a in G has only one inverse in G. **(6 Marks)**

(b) Show that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all a, b in G. **(6 Marks)**

(c) If f is a homomorphism from a commutative semigroup $(S, *)$ onto a semigroup $(T, *')$, then $(T, *')$ is also commutative. **(8 Marks)**

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CS34

USN

NEW SCHEME

Third Semester B.E. Degree Examination, July 2006

CS / IS

Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note: 1. *Answer any FIVE full questions.*
2. *Figures to the right indicate full marks.*

1 a. Explain the laws of set theory. **(04 Marks)**

b. For any two sets A and B, prove the following :

$$A-(A-B)=A\cap B$$ **(04 Marks)**

c. Explain what do you mean by a probability of an event E, and then solve the following problem.
If one tosses a fair coin four times, what is the probability of getting two heads and two tails? **(04 Marks)**

d. Solve the following problem:

i) By the principle of induction, prove that

$$1^2+2^2+3^2+\ldots\ldots+n^2=\frac{1}{6}n(2n+1)(n+1)$$

ii) Find an explicit formula for the sequence defined,
$C_n = 3C_{n-1} - 2C_{n-2}$
with the initial conditions $C_1 = 5$ and $C_2 = 3$. **(08 Marks)**

2 a. Discuss, the basic connectives that are used in logic. **(04 Marks)**

b. Given p and q as statements, explain the following terms.

i) Conjunction ii) Disjunction
iii) Logically equivalence iv) Tautology **(04 Marks)**

c. Solve the following :

i) Show that
$(p\vee q)\leftrightarrow(q\vee p)$ is a tautology.

ii) Consider the following argument :
I will get grade A in this course or I will not graduate.
If I do not graduate, I will join army.
I got grade A.
Therefore, I will not joint the army.
Is this valid argument? Prove using rules of inferences.

iii) Given R(x, y) : $x + y$ is even and the variables x and y represent integers. Write an English sentence corresponding to each of the following:

- $\forall x\exists yR(x,y)$
- $\exists x\forall yR(x,y)$

(12 Marks)

Contd....2

3 a. Let A = {a, b, c, d}, and let R be the relation on A and has the matrix

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Construct the degree of R and list the in-degrees and out-degrees of all vertices. **(04 Marks)**

b. Define equivalence relation R on the given set A.
Let A = Z and let
$R = \{(a,b) \in A \times A \,/\, a \equiv r(\text{mod } 2) \text{ and } b \equiv r(\text{mod } 2)\}$ and Let R be a equivalence relation defined for the above. Then determine A/R. **(04 Marks)**

c. What is Poset? Discuss at least with four examples.
And let A = {1, 2, 3, 4, 12}. Consider the partial order of divisibility on A.
Then draw the Hasse diagram of the poset (A, ≤). **(06 Marks)**

d. If the poset is to be a Lattice, then what are the properties it should satisfy?
Hasse diagrams for two posets are given in the following diagrams. Determine with reasons whether or not they an Lattices. **(06 Marks)**

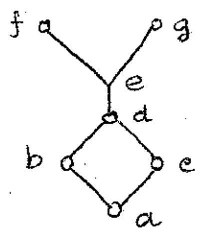

Fig Q3 (d)-1

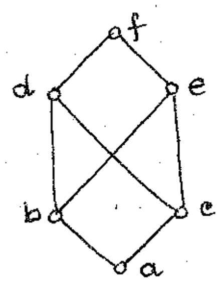

Fig Q3 (d)-2

4 a. Let $A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3\}$, $C = \{c_1, c_2\}$ and $D = \{d_1, d_2, d_3, d_4\}$. Consider the following four functions, from $A \to B$, $A \to D$, $B \to C$ and $D \to B$ respectively.

i) $f_1 = \{(a_1, b_2), (a_2, b_3), (a_3, b_1)\}$
$f_2 = \{(a_1, d_2), (a_2, d_1), (a_3, d_4)\}$
$f_3 = \{(b_1, c_2), (b_2, c_2), (b_3, c_1)\}$
$f_4 = \{(d_1, b_1), (d_2, b_2), (d_3, b_1)\}$

Determine whether or not each function is one-one, onto and every where defined. **(08 Marks)**

b. Write a note on Hashing functions. **(04 Marks)**

c. Discuss, in detail, the various functions that are used in computer science. **(08 Marks)**

5 a. Let A = {1, 2, 3, 4, 5, 6}
Compute (4, 1, 3, 5) ∘ (5, 6, 3) and (5, 6, 3) ∘ (4, 1, 3, 5)
Show that the above two products are neither equal nor a cycle. **(04 Marks)**

b. Let A = $\{a_1, a_2, \ldots\ldots\ldots a_n\}$ be a finite set with n-elements, $n \geq 2$. Then prove that there are $\frac{n!}{2}$ even permutations and $\frac{n!}{2}$ odd permutations. **(06 Marks)**

c. Discuss about the composite functions and hence solve the following problem.
Let A = {1, 2, 3, 4}, B = {a, b, c} and C = {w, x, y, z}
With $f : A \to B$ and $g : B \to C$ and is given by
$f = \{(1, a), (2, a), (3, b), (4, c)\}$
$g = \{(a, x), (b, y), (c, z)\}$ then find $g \circ f$. **(04 Marks)**

d. Given A = {1, 2, 3}, write all the permutations of A.
Then compute i) P_4^{-1} and ii) $P_3 \circ P_2$
And show also that $P_3 \circ P_2 = P_5$ **(06 Marks)**

6 a. Write a short note on Binary operation and their properties. **(06 Marks)**
b. Define a group, and also explain what is Abelian group? **(04 Marks)**
c. What is cyclic group? Explain, and hence, show that the group (G, *) whose multiplication table is as given below is cyclic. **(06 Marks)**

*	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	d	e	f	a
c	c	d	e	f	a	b
d	d	e	f	a	b	c
e	e	f	a	b	c	d
f	f	a	b	c	d	e

d. Explain Lagrange's theorem.
If G is a group of order n, and $a \in G$,
Prove that $a^n = e$ **(04 Marks)**

7 a. Define the following terms with respect to coding theory.
i) Parity check code
ii) Hamming distance
iii) Group code
iv) Generator matrix **(08 Marks)**

b. The parity-check matrix for an encoding function E : Z_2^3-$7Z_2^6$ is given by,

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) Determine the associated generator matrix.
ii) Does this code correct all single errors in transmission? **(04 Marks)**

c. Given R as a non-empty set with two closed operations, denoted by + and •, then explain when (R, +, •) will be a ring. **(04 Marks)**
d. Explain, briefly the encoding and de-coding of a message. **(04 Marks)**

8 a. If A and B are two sets, then define the following term (Use Venn diagrams)
i) Union of two sets. iii) Intersection of the two sets.
ii) Symmetric difference iv) Compliment of B with respect to A. **(06 Marks)**

b. Explain when a relation R on a set A is
i) Reflexive iii) Symmetric
ii) Anti-symmetric iv) Transitive **(06 Marks)**
with a simple example for each.

c. Write a short note on "Linearly ordered set". **(04 Marks)**

d. Write the permutation $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 6 & 5 & 2 & 1 & 8 & 7 \end{pmatrix}$
of the set A = {1, 2, 3, 4, 5, 6, 7, 8} as a product of disjoint cycles. **(04 Marks)**

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NEW SCHEME

CS34

Reg. No.

Third Semester B.E. Degree Examination, January/February 2006

Computer Science/Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.) (Max.Marks : 100

Note: ***1. Answer any FIVE full questions.***
2. All questions carry equal marks.

1. (a) Determine the sets A and B, given that $A - B = \{1,2,4\}$, $B - A = \{7,8\}$, and $A \cup B = \{1,2,4,5,7,8,9\}$ **(4 Marks)**

(b) Prove that : $\overline{A \triangle B} = \overline{A} \triangle B = A \triangle \overline{B}$ **(6 Marks)**

(c) The freshman class of a private engineering college has 300 students. It is known that 180 can program in PASCAL, 120 in FORTRAN, 30 in C^{++}, 12 in PASCAL and C^{++}, 18 in FORTRAN and C^{++}, 12 in PASCAL and FORTRAN, and 6 in all three languages. If two students are selected at random, what is the probability that they can i) both program in PASCAL ? ii) both program only in Pascal. **(6 Marks)**

(d) Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(4 Marks)**

2. (a) If statement q has the truth value 1, determine all truth value assignments for the primitive statements p, r and s for which the truth vaue of the statement.

$$[q \rightarrow \{\neg\, p \vee r) \wedge \neg\, s\}] \wedge \{\neg\, s \rightarrow (\neg\, r \wedge q)\}$$ **(4 Marks)**

(b) Define tautology. Prove that, for any propositions p, q, r, the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)]$ is a tautology. **(6 Marks)**

(c) Prove the following logical equivalences without using truth tables :

i) $p \vee [p \wedge (p \vee q)] \Longleftrightarrow p$

ii) $\neg\, [\neg\, \{p \vee q) \wedge r\} \vee \neg\, q] \Longleftrightarrow q \wedge r$ **(6 Marks)**

(d) Test whether the following argument is valid :

If interest rates fall, then the stock market will rise. The stock market will not rise. Therefore the interest rates will not fall. **(4 Marks)**

3. (a) Consider the following open statements with the set of all real numbers as the universe

$$p(x) : x \geq 0,\ \ q(x) : x^2 \geq 0$$
$$r(x) : x^2 - 3x - 4 = 0,\ s(x) : x^2 - 3 > 0,$$

then find the truth values of the following statement.

i) $\exists\, x\, [p(x) \wedge r(x)]$

ii) $\forall\, x\, [p(x) \rightarrow q(x)]$

iii) $\forall\, x\, [q(x) \rightarrow s(x)]$ **(6 Marks)**

Contd.... 2

(b) Write down the following propositions in symbolic form, and find its negation.

i) For all integers n, if n is not divisible by 2, then n is odd.

ii) All integers are rational numbers and some rational numbers are not integers. **(7 Marks)**

(c) For the universe of all people, find wherher the following is a valid argument :

All mathematics professors have studied calculus
Ramanujan is a mathematics professor
Therefore Ramanujan has studied calculus. **(7 Marks)**

4. (a) Define cartesian product of two sets. For any three non-empty sets A, B, C prove that $A \times (B - C) = (A \times B) - (A \times C)$ **(6 Marks)**

(b) Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if a is a multiple of b. Write down the relation matrix $M(R)$ and draw its digraph. **(6 Marks)**

(c) If $A = \{1, 2, 3, 4\}$ and R, S are relations on A defined by $R = \{(1,2), (1,3), (2,4), (4,4)\}$, $S = \{(1,1)\ (1,2)\ (1,3)(1,4), (2,3)\ (2,4)\}$ find RoS, SoR, R^2 and S^2. **(4 Marks)**

(d) Let $A = \{a, b, c, d, e\}$. Consider the partition $P = \{\{a, b\}, \{c, d\}\{e\}\}$ of A. Find the equivalance relation inducing this partition. **(4 Marks)**

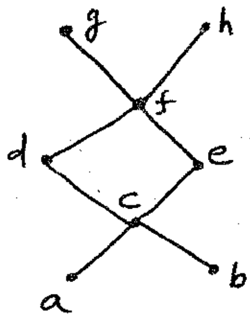
5. (a) Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on $A \times A$ by $(x_1, y_1)\ R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Verify that R is an equivalence relation on $A \times A$

ii) Determine the equivalence classes $[(1,3)]$, $[(2,4)\ and\ [1,1]]$ **(6 Marks)**

(b) Define a partially ordered set. If R is a relation on the set $A = \{1, 2, 3, 4\}$ defined by xRy if x divides y, prove that (A, R) is a poset. Draw its Hasse diagram. **(7 Marks)**

(c) Let A be a set and $B \subseteq A$. Define i) Least upper bound of B ii) Greatest lower bound of B. Consider the poset whose Hasse diagram is shown below
Find LUB and GLB of the set $B = \{c, d, e\}$



(7 Marks)

6. (a) Prove that, a function $f : A \longrightarrow B$ is invertible if and only if it is one-one and onto. **(6 Marks)**

(b) Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set $A = \{1, 2, 3, 4, 5, 6\}$.

i) Write p as a product of disjoint cycles

ii) Compute p^{-1}

iii) Compute p^2 and p^3

iv) Find the smallest integer k such that $p^k = I_A$ **(7 Marks)**

(c) Define characteristic funtion. For any sets A, B contained in a universal set U, prove that:

i) $f_{\overline{A}}(x) = 1 - f_A(x)$

ii) $f_{A \cap B}(x) = f_A(x) \cdot f_B(x)$ **(7 Marks)**

7. (a) Define an abelian group. Prove that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$. **(6 Marks)**

(b) Prove that, if H is a non empty subset of a group G, then H is a subgroup of G if and only if

i) for all $a, b \in H,\ ab \in H$ and

ii) for all $a \in H,\ a^{-1} \in H$ **(6 Marks)**

(c) Define left and right cosets. State and prove Lagrange's theorem. **(8 Marks)**

8. (a) Define a ring. If R is a ring with unity and $a,\ b$ are units in R, prove that ab is a unit in R and that $(ab)^{-1} = b^{-1}a^{-1}$. **(6 Marks)**

(b) If $f : G \longrightarrow H,\ \ g : H \longrightarrow K$ are homomorphisms, prove that the composite function $gof : G \longrightarrow K$, where $(gof)\ (x) = g(f(x))$, is a homomorphism. **(6 Marks)**

(c) Define a group code. Let $E : {Z_2}^m \longrightarrow {Z_2}^n$ be an encoding function given by a generator matrix G or the associated parity - check matrix H. Then prove that $C = E({Z_2}^m)$ is a group code. **(8 Marks)**

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CS32

USN

OLD SCHEME

Third Semester B.E. Degree Examination, July 2006
Computer Science and Engineering
Discrete Mathematical Structure

Time: 3 hrs.] [Max. Marks:100

Note: 1. ***Answer any FIVE full questions.***

1 a. Define a set, subset, empty set, universal set, power set and symmetric difference. Give one example for each. (06 Marks)

b. Prove the following:

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii) $A - B = A - (A \cap B)$

and verify the above using Venn diagram also. (08 Marks)

c. In a language survey of 100 students, it was found that 32 know Tamil, 20 know Kannada, 45 know Telugu, 50 know Tamil and Telugu, 7 know Tamil and Kannada, 10 know Kannada and Telugu, 30 don't know any of the three languages. Determine

i) The number of students who know all the three languages.

ii) The number of students who know exactly one of the three languages. (06 Marks)

2 a. Explain Tataulogy and contradiction, check whether the following is tataulogy contradiction.

i) $p \vee (q \cup r) \rightarrow (p \cap q) \vee (p \cap r)$

ii) $p \cap (q \cup r) \rightarrow (p \vee q) \cap (p \vee r)$ (08 Marks)

b. Explain PDNF and PCNF with examples. (08 Marks)

c. Find DNF and CNF for the expression,

$\neg(p \vee q) \rightleftarrows (p \cap q)$ (04 Marks)

3 a. State and explain pigeon hole principle. (07 Marks)

b. Prove by mathematical induction,

$$1+2+3+\dots\dots\dots+n = \frac{n(n+1)}{2}$$ (08 Marks)

c. In how many ways a committee consisting of three men and two women be chosen from seven men and five women. (05 Marks)

4 a. If A and B are finite sets, prove that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ (04 Marks)

b. Define Cartesian product, Partition, and relation with examples. (06 Marks)

c. Define various properties of relation. (04 Marks)

d. Let $A = \{1, 2, 3, 4, 6\}$ and let R be the relation on A defined by "x divides y".

i) Write R as a set of ordered pairs

ii) Draw the directed graph.

iii) Find the inverse relation R^{-1} of R. (06 Marks)

Contd.... 2

5 a. Prove that the congruence relation is an equivalence relation. (06 Marks)
b. Let $A = \{1, 2, 3, 4\}$, $R=\{(1, 2) (2, 3) (3, 4) (2, 1)\}$, find the transitive closure of R. (08 Marks)
c. Let R be an equivalence relation on a set S, then prove that the quotient S/R is a partition of S. (06 Marks)

6 a. Define function, one to one, onto and give one example for each. (06 Marks)
b. Let $A = \{1, 2, 3\}$ then write all permutation of A. (08 Marks)
c. Let $f: R \rightarrow R$ be defined by $f(x) = 2x-3$ now f is one to one and onto: hence f has inverse function f^{-1}. Find f^{-1}. (06 Marks)

7 a. Define Poset with an example. (04 Marks)
b. Define maximal element, minimal element, greatest element and least element with examples. (08 Marks)
c. Let $S = \{a, b, c\}$ and $A = P(S)$. Write the powerset of S and draw the Hase diagram. (08 Marks)

8 a. Define semigroup, monoid, group and abelian group with examples. (10 Marks)
b. Prove that the set $G = \{0, 1, 2, 3, 4\}$ is a finite abelian group of order 5 under addition modulo 5 as composition. (10 Marks)

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NEW SCHEME

CS34

USN

Third Semester B.E. Degree Examination, July/August 2005

Computer Science /Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) Determine the sets A and B, given that

$A - B = \{1,3,7,11\}$, $B - A = \{2,6,8\}$ and $A \cap B = \{4,9\}$ (4 Marks)

(b) prove that :

$A \Delta B = (B \cap \overline{A}) \cup (A \cap \overline{B}) = (B - A) \cup (A - B)$ (6 Marks)

(c) A survey of 500 televisions viewers of sports channel produced the following information : 285 watch cricket, 195 watch hockey, 115 watch foot ball, 45 watch cricket and foot ball, 70 watch cricket and Hockey, 50 watch Hockey and foot ball and 50 do not watch any of the three kinds of games.

i) How many viewers in the survey watch all three kinds of games?

ii) How many viewers watch exactly one the sports. (6 Marks)

(d) By mathematical induction, prove that $\lfloor\underline{n} \geq 2^{n-1}$ for all integers $n \geq 1$. (4 Marks)

2. (a) Define tautology and contradiction of a compound proposition. Prove that, for any propositions p, q, r the compound proposition :

$[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology. (8 Marks)

(b) Prove that $[(\neg p \vee q) \wedge (p \wedge (p \wedge q))] \Leftrightarrow p \wedge q$

Hence deduce that $[(\neg p \wedge q) \vee (p \vee (p \vee q))] \Leftrightarrow p \vee q$ (6 Marks)

(c) Define the following :

Rule of syllogism ii) Modus pones iii) Modus tollens.

Test whether the following argument is valid : If I drive to work, then I will arrive tired. I am not tired (when I arrive at work) Hence, I donot drive to work. (6 Marks)

3. (a) Define : i) Open sentences ii) Quantifiers.

Write down the following proposition in symbolic form, and find is negation :

"If all triangles are right-angled, then no triangle is equiangular". (7 Marks)

(b) Find whether the following is a valid argument for which the universe is the set of all students.

No engineering student is bad in studies.
Ram is not bad in studies.
Therefore, Ram is an engineering student. (7 Marks)

Contd.... 2

(c) Give :

i) A direct proof

ii) An indirect proof and

iii) Proof by contradiction, for the following statement :

"If n is an odd integer, then $(n+9)$ is an even integer". (6 Marks)

4. (a) Let $A=\{1,2,3\}$ and $B=\{1,2,3,4\}$. The relations R and S from A to B are represented by the following matrices. Determine the relations $\overline{R}$, $R\cup S$ and $R\cap S$ and their matrix representations.

$$M(R)=\begin{bmatrix}1&0&1&0\\0&0&0&1\\1&1&1&0\end{bmatrix},\ M(S)=\begin{bmatrix}1&1&1&1\\0&0&0&1\\0&1&0&1\end{bmatrix},$$ (7 Marks)

(b) Let $A=\{1,2,3,4\}$ and R a relation on A defined by $R=\{(1,2),\ (1,3),(2,4),(3,2),(3,3),(3,4)\}$. Find R^2 and R^3. Write down the graphs of R, R^2 and R^3. (6 Marks)

(c) Define the following with one example each :

i) Reflexive relation

ii) Symmetric relation

iii) Antisymmetric relation.

let R and S be relations on a set A. If R and S are symmetric, prove that $R\cap S$ also is symmetric. (7 Marks)

5. (a) Define an equivalence relation with an example. Let $A=\{1,2,3,4,5\}$. Define a relation R and $A\times A$ by $(x_1,y_1)R(x_2,y_2)$ if $x_1+y_1=x_2+x_2$. Verify that R is an equivalence relation on $A\times A$. (6 Marks)

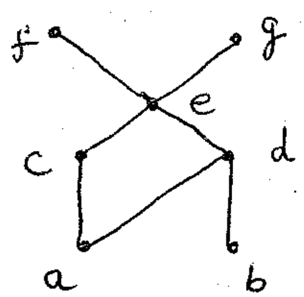
(b) Define a partial order on a set A with an example. let $A=\{1,2,3,4,6,12\}$. On A, define the relation R by aRb if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation. (7 Marks)

(c) Let A be a set and $B\subseteq A$. Define

i) Least upper bound $(L\cup B)$ of B.

ii) Greatest lower bound (GLB) of B.

Consider the poset whose Hasse diagram is shown below. Find $L\cup B$ and GLB of the set $B=\{c,d,e\}$. (7 Marks)



6. (a) Define a function from a set A to the set B. Distinguish between a relation and a function. Let A and B be finite sets with $|A| = m$ and $|B| = n$. Find how many functions are possible from A to B? If there are 2187 functions from A to B and $|B| = 3$, what is $|A|$? **(8 Marks)**

   (b) Define stirling number of the second kind. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{w, x, y, z\}$. Find the number of onto functions from A to B. **(6 Marks)**

   (c) Define :

   Permutation function ii) Characteristic function.

   Given $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$, compute p^{-1} and p^{-2}. **(6 Marks)**

7. (a) Define an Abelian group. Let $(G, *)$ be the set of all non-zero real numbers and let $a * b = \frac{1}{2}ab$. Show that $(G, *)$ is an Abelian group. **(8 Marks)**

   (b) Define a subgroup. Let G be a group and $G_1 = \{x \in G | xy = yx \text{ for all } y \in G\}$. Prove that G_1 is a subgroup of G. **(6 Marks)**

   (c) Define a cyclic group with an example. Prove that every cyclic group is abelian. **(6 Marks)**

8. (a) State and prove Lagrange's theorem. **(6 Marks)**

   (b) Define homomorphism and isomorphism in a group. let f be a homomorphism from a group G_1 to a group G_2. Prove that

   i) If e_1 is the identify in G_1 and e_2 is the identity in G_2, then $f(e_1) = e_2$.

   ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G_1$. **(8 Marks)**

   (c) The generator matrix for an encoding function $E : Z_2^3 \to Z_6^6$ is given by

   $$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

   Find the code words assigned to 110 and 010. Also obtain the associated parity check matrix. **(6 Marks)**

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NEW SCHEME

CS3

USN

Third Semester B.E. Degree Examination, January/February 2005

Computer Science /Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. Missing data may be suitably assumed.

1. (a) In a survey of 120 passengers, an airline found that 48 enjoyed wine with their meals, 78 enjoyed mixed drinks, 66 enjoyed iced tea. In addition, 36 enjoyed any given pair of these beverages and 24 enjoyed them all. If two passengers are selected at random from the survey sample of 120, what is the probability that they both want only iced tea with their meals? (7 Marks)

   (b) For all $n \in Z^+$, show that if $n \geq 24$ then n can be written as a sum of 5's and / or 7's. (6+1 Marks)

   (c) Show that $\overline{A \cup B} = \overline{A} \cap \overline{B}$ where $A, B \subset$ U. (6 Marks)

2. (a) If a band could not play rock music or the refreshments were not delivered on time, then new year's party would have been canceled and Alicia would have been angry. If the party were canceled, then refunds would have to be made. No refunds were made therefore the band could play rock music. Convert the given argument into symbolic form by using the following statement assignments.

   p : the band could play rock music
   q : the refreshments were delivered on time
   r : the new year's party was canceled
   s : Alicia was angry
   t : refunds had to be made

   Establish the validity of the arguments using rules of inference. (8 Marks)

   (b) Verify that $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is tautology. (4 Marks)

   (c) State any four rules of inference and give an example for each. (8 Marks)

3. (a) Simplify the following compound statements using laws of logic. Mention the reasons :

   i) $(p \vee q) \wedge \neg(\neg p \vee q)$

   ii) $\neg[\neg\{(p \vee q) \wedge r\} \vee \neg q]$ (4+4=8 Marks)

   (b) Let the universe comprise of all integers

   i) Given $p(x) : x$ is odd ; $q(x) : x^2 - 1$ is even

   Express the statement "If x is odd then $x^2 - 1$ is even" in symbolic form using quantifiers and negate it.

   ii) If $r(x) : 2x + 1 = 5; s(x) : x^2 = 9$ are open sentenced, obtain the negation of the quantified statement $\exists\, x[r(x) \wedge s(x)]$ (4+4=8 Marks)

   (c) For any two statements p and q, prove that

   $\neg(p \uparrow q) \Longleftrightarrow (\neg p \downarrow \neg q)$. (4 Marks)

4. (a) If R is an equivalence relation on A and $x, y \in A$ then show that the following statements are equivalent

i) $x \in [x]$ ii) xRy if f $[x] = [y]$

iii) $[x] = [y]$ or $[x] \cap [y] = \phi$ (2+4+4=10 Marks)

(b) Let T be the set of all triangles. Define a relation R on T by $t_1 R t_2$ if t_1 and t_2 have an angle of the same measure. Verify whether R is an equivalence relation. (4 Marks)

(c) Draw the digraph $G = (V, E)$ where $V = \{a, b, c, d, e, f\}$ and $E = \{(a,b), (a,d), (b,c), (b,e), (d,b), (d,e), (e,c), (e,f), (f,d)\}$. (4 Marks)

(d) If $U = \{1,2,3,4,5\}$, $A = \{1,2,3\}$, $B = \{2,4,5\}$ determine the number of relations from A to B. (2 Marks)

5 (a) Verify that (A, R) is poset and draw its Hasse diagram where $u = \{1,2,3\}, A = p(u)$ and R is a subset relation on A. (4+4=8 Marks)

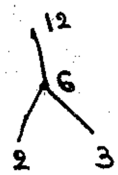
(b) Topolgically sort (A, R) where

$A = \{2,3,5,7,11,6,12,35,385\}$

$R = \{(x,y) \mid x \text{ exactly divides } y\}$ (6 Marks)

(c) Prove or disprove the following statement 'If (A, R) is lattice then it is a total order. (2 Marks)

(d) Given $R = \{2,3,6,12\}$ and relation defined on A by Hasse diagram, find $lub\{2,3\}, glb\{2,3\}$, $lub\{2,12\} \& gld\{6,12\}$. Is (A, R) a lattice? If not give reasons. (4 Marks)



6. (a) Let $A = \{1,2,3,4,5,6\}$ show that

$(4,1,3,5)\ 0\ (5,6,3) \neq (5,6,3)\ 0\ (4,1,3,5)$ (6 Marks)

(b) Let $f, g, h : R \to R$ where $f(x) = x^2; g(x) = x+5; h(x) = \sqrt{x^2+2}$ show that $(hog)of = ho(gof)$. (4 Marks)

(c) A function $f : A \to B$ is invertible iff it is one-to-one and onto. Prove this statement. (8 Marks)

(d) Given $A = \{1,2,3,4\}$ and $f : A \to A$ defined by $f = \{(1,2),(2,2),(3,1),(4,3)\}$, find f^2. (2 Marks)

7. (a) For any group G prove that G is abelian iff $(ab)^2 = a^2b^2\ \forall a, b \in G$. (8 Marks)

(b) Define left coset and right coset. State and prove Lagrange's theorem. (2+2+4=8 Marks)

(c) If R is a ring with unity and a, b are units of R, prove that ab is a unit of R and $(ab)^{-1} = b^{-1}a^{-1}$. (4 Marks)

8. (a) Define the binary operation 0 on Z by

$x\ 0\ y = x + y + 1$.

Verify that $(z, 0)$ is an abelian group. (8 Marks)

(b) Let $(G, O) \& (H, *)$ be groups with respective identities $e_G \& e_H$. Iff : $G \to H$ is homomorphism, then show that

i) $f(e_G) = e_H$ ii) $f(a^1) = [f(a)]^{-1}\ \forall a \in G$ (4+4=8 Marks)

(c) Prove that in a group code, the minimum distance between distinct code words is the minimum of the weights of the non-zero elements of the code. (4 Marks)

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CSI32

USN

Third Semester B.E. Degree Examination, January/February 2005

Computer/Information Science and Engineering

(Old Scheme)

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE questions.*
2. All questions carry equal marks.

1. (a) Define the following with examples :

i) Symmetric difference

ii) Complement of sets

iii) Power set

(6 Marks)

(b) Show that for any two sets $A, B, \& C$

i) $A-(A\cap B)=A-B$

ii) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ (6 Marks)

(c) In a survey of 260 college students, the following data were obtained.

64 had taken mathematics course
94 had taken computer science course
58 had taken business course
28 had taken both mathematics & business course
26 had taken both mathematics and computer science course
22 had taken both computer science and business course
14 had taken all three types of courses

a) How many students were surveyed who had taken none of the three types of courses?

b) Of the students surveyed, how many had taken only a computer science course? (8 Marks)

2. (a) Show that n^3+2n is divisible by 3. (4 Marks)

(b) Solve recurrence relation $a_n = a_{n-1} + n$ with $a_1 = 4$ (6 Marks)

(c) Show that

i) $P \longrightarrow (Q \longrightarrow R) \Longleftrightarrow P \longrightarrow (\Gamma Q \cup R) \Longleftrightarrow (P \cap Q) \longrightarrow R$

ii) $(\Gamma P \cap (\Gamma Q \cap R)) \cup (Q \cap R) \cup (P \cap R) \Longleftrightarrow R$ (6 Marks)

(d) Define tautology and contradiction with example. (4 Marks)

3. (a) Show that, if any 11 numbers are chosen from the set $\{1,2,...20\}$, then one of them will be a multiple of another. (6 Marks)

(b) In how many ways can six men and six women be seated in a row, if.

i) any person may sit next to any other

ii) men and women must occupy alternate seats. (6 Marks)

Contd.... 2

CSI32

USN

Third Semester B.E. Degree Examination, July/August 2005

Computer/Information Science and Engineering

(Old Scheme)

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) If A and B are any two finite sets then prove that $A-(A-B)=A\cap B$. (6 Marks)

   (b) Define and give the examples for each

   i) Power set ii) Finite set iii) Null set (6 Marks)

   (c) How many different eleven person cricket team can be formed each containing 5 female members from an available set of 20 females and 6 male members from an available set of 30 males. (4 Marks)

   (d) Define extended Pigeon hole principle. Show that if any 30 students are selected, then you may choose a subset of 5 so that all 5 were born on the same day of the week. (4 Marks)

2. (a) Prove by mathematical induction

   $$1^3+2^3+3^3+.....+n^3=\frac{n^2(n+1)^2}{4}$$ (7 Marks)

   (b) Define :

   i) Tautology

   ii) Contradiction

   iii) Contingence

   Give an examples for each. (6 Marks)

   (c) Find the negation of the following statement

   "All odd numbers are not prime numbers and some prime numbers are even". (7 Marks)

3. (a) List and explain the properties of relations with examples. (6 Marks)

   (b) Find the domain, range, matrix and digraph of the relation R

   $A=\{1,2,3,4,6\}=B$ aRb iff a is a multiple of b. (8 Marks)

   (c) Let $A=B=\{1,2,3,4\}$ and R is the relation on A given by

   $R=\{(1,3),(2,1),(2,3),(3,1),(3,2),(3,4)(4,2)\}$

   Compute

   i) R^{-1}

   ii) Digraph of R^{-1}

   iii) Matrix of $\overline{R}$ (6 Marks)

Contd.... 2

4. (a) If $f : A \rightarrow B$ is one-to-one and onto function then prove that $f^{-1} : B \rightarrow A$ is also one-to-one and onto. **(6 Marks)**

(b) Let $A = \{1, 2, 3, 4, 5, 6\}$ compute

i) (4, 1, 3, 5) 0 (5, 6, 3)

ii) (5, 6, 3) 0 (4, 1, 3, 5) **(8 Marks)**

(c) Prove that the product of two even permutations is even. **(6 Marks)**

5. (a) Define :

i) POSET ii) LUB
iii) GLB iv) Lattice
v) Hasse diagram

(10 Marks)

(b) Is the POSET $A = \{2, 3, 6, 12, 24, 36, 72\}$ under the relation of divisibility a Lattice. **(6 Marks)**

(c) State and explain different properties of Lattice. **(4 Marks)**

6. (a) Show that if n is a positive integer and $\frac{P^2}{n}$, where P is a prime number, then D_n is not a Boolean Algebra. **(8 Marks)**

(b) Let $P(x, y, z) = ((x \wedge y) \vee (y \wedge \overline{z}))$, find the truth table for the corresponding function $f : A_3 \rightarrow A$. **(4 Marks)**

(c) Define :

i) Groups

ii) Semi groups

Give examples for each **(8 Marks)**

7. (a) Let $(S, *)$ and $(T, *)$ be monoids with identicle e and e^1 respectively. Prove that if $f : S \rightarrow T$ be an isomorphism then $f(e) = e^1$. **(8 Marks)**

(b) Let G be group. Prove that each element a in G has only one inverse in G. **(6 Marks)**

(c) Let G be a group and a& b be elements of G. Prove that

i) $(a^{-1})^{-1} = a$

ii) $(ab)^{-1} = a^{-1}b^{-1}$ **(6 Marks)**

8. Write short notes on:

a) Watchall Algorithms

b) Characteristic function

c) Monoids

d) Recurance relations **(4×5=20 Marks)**

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CSI32

USN

Third Semester B.E. Degree Examination, January/February 2004

Computer/Information Science and Engineering

(Old Scheme)

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) Show that for any two sets A and B

i) $P(A) \cup P(B) \subseteq P(A \cup B)$

ii) $P(A) \cap P(B) = P(A \cap B)$

(5 Marks)

(b) Define the characteristic function of a set and prove that

$f_{A \oplus B} = f_A + F_B - 2f_A f_B$.

(5 Marks)

(c) Let $A = \{+, \chi, a, b\}$. Then show that the following expressions are regular over A

i) $a + bx(a^* V b)$.

ii) $((a^* bV+)^* \chi ab^*)$.

(5 Marks)

(d) Prove by mathematical induction that

$1 + 2^n < 3^n$ for $n \geq 2$.

(5 Marks)

2. (a) State pigeonhole principle. Prove that if any 14 numbers from 1 to 25 are choosen then one of them is a multiple of another.

(5 Marks)

(b) Five fair coins are tossed and results are recovered.

i) How many different sequences of head and tail are possible?

ii) How many of the sequences in (i) have exactly one head recorded?

iii) How many of the sequences in (i) have exactly three head recorded? **(5 Marks)**

(c) Currently, telephone area codes are three digit numbers, whose middle digit must be 0 or 1. Codes whose last two digits are 1's are being used for other purposes. With these conditions, how many area codes are available?

(5 Marks)

(d) If the characteristic equation $x^2 - r_1 x - r_2 = 0$ of the relation $a_n = r_1 a_{n-1} + r_2 a_{n-2}$ has a repeated root s, then show that $a_n = (u + vn)s^n$ is the explicit formula for the sequence, where u & v depend on the initial conditions.

(5 Marks)

3. (a) Define :

i) predicate; ii) universal quantifier iii) essential quantifier.

Let $A = \{1, 2, 3, 4, 5\}$. Determine the truth value of each of the following statements.

i) $\exists\, x \in A(x + 8 = 14)$ ii) $(\forall x \in A)(x + 8 < 14)$.

(5 Marks)

(b) Let 'n' be an integer. Prove that if n^2 is odd then 'n' is odd.

(5 Marks)

(c) Prove the implication property

$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$.

(5 Marks)

(d) List all partitions of $A = \{a, b, c, d\}$.

(5 Marks)

Contd.... 2

Page No. 1

NEW SCHEME

CS34

USN

Third Semester B.E. Degree Examination, January/February 2004

Computer Science /Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*
All questions carry EQUAL marks.

1. (a) How many rows are needed for the truth table of the compound statement

$$(p \vee \neg q) \leftrightarrow [(\neg r \wedge s) \rightarrow t]$$

where p, q, r, s and t are primitive statements? (2 Marks)

(b) Define tantology. Prove that

$$((P \vee Q) \wedge \neg (\neg P \wedge (\neg Q \vee \wedge R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$$

is a tantology. (8 Marks)

(c) For any two statements p and q, prove that

$$\neg (p \uparrow q) \Leftrightarrow (\neg p \downarrow \neg q)$$

(4 Marks)

(d) State whether the arguments given below are valid or not. If an argument is valid, identify the tantology or tantologics on which it is based.

i) If I drive to work, then I will arrive tired.
I am not tired when I arrive at work.
Therefore I do not drive to work.

ii) I will become famous or I will not become a musician.
I will become a musician.
Therefore I will become famous.

(6 Marks)

2. (a) For the universe of all integers, let p(x), q(x), r(x), s(x) and t(x) denote the following open statements.

$p(x) : x > 0$

$q(x) : x$ is even

$r(x) : x$ is a perfect square

$s(x) : x$ is divisible by 3

$t(x) : x$ is divisible by 7

Write the following statements in symbolic form.

i) At least one integer is even.

ii) There exists a positive integer that is even.

iii) If x is even, then x is not divisible by 3.

iv) No even integer is divisible by 7.

v) There exists an even integer divisible by 3.

(5 Marks)

Contd.... 2

(b) Let $p(x), q(x)$ and $r(x)$ denote the following open statements.

$p(x) : x^2 - 7x + 10 = 0$

$q(x) : x^2 - 2x - 3 = 0$

$r(x) : x < 0$

Determine the truth or falsity of the following statements when the universe contains only the integers 2 and 5. If a statement is false, provide a counter example or explanation.

i) $\forall x[p(x) \rightarrow \urcorner r(x)]$

ii) $\forall x[q(x) \rightarrow r(x)]$

i) $\exists x[q(x) \rightarrow r(x)]$

i) $\exists x[p(x) \rightarrow r(x)]$ (4 Marks)

(c) Write the negation of each of the following statements. For i) and ii), the universe consists of all integers and for iii), the universe consists of all real numbers.

i) For all integers n, if n is not divisible by 2, then n is odd.

ii) If k, m, n are any integers where $k - m$ and $m - n$ are odd, then $k - n$ is even.

iii) For all real numbers x, if $| x - 3 | < \urcorner$,

then $-4 < x < 10$ (3 Marks)

(d) For each of the following statements, provide an indirect proof by stating and proving the contrapositive of the given statement.

i) For all integers k and l, if kl is odd, then both k, l are odd.

ii) For all integers k and l, if $k + l$ is even, then k and l are both even or both odd. (8 Marks)

3. (a) For any three sets A, B and C prove that

$$(A - B) - C = A - (B \cup C) = (A - C) - (B - C)$$ (8 Marks)

(b) Thirty cars are assembled in a factory. The options available are a transistor, an air conditioner and power windows. It is known that 15 of the cars have transistors 8 of them have conditioners and 6 of them have power windows. Moreover, 3 of them have all three options. Determine atleast how many cars do not have any options at all. (6 Marks)

(c) For all positive integers n, prove that if $n \geq 24$, then n can be written as a sum of 5s and / or 7s. (6 Marks)

4. (a) Ackermann's function A(m,n) is defined recursively as follows:

$A(0, n) = n + 1 (n \geq 0)$

$A(m, 0) = A(m - 1, 1) (m > 0)$

$A(m, n) = A(m - 1), A(m, n - 1)) \quad (m, n > 0)$

Prove that $A(1, n) = n + 2$ for all $n \in N$ (6 Marks)

(b) Define $S(m, n)$, sterling number of the second kind. If m and n are positive integers with $1 < n \leq m$, then prove that

$$S(m + 1, n) = S(m, n - 1) + nS(m, n)$$ (8 Marks)

(c) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions. If $gof : A \rightarrow C$ is onto, prove that g is onto and if $gof : A \rightarrow C$ is one-to-one, then prove that f is one-to-one.

Contd.... 3

(6 Marks)

5\. (a) A relation R on a set A is said to be irreflexive if $(x, x) \notin R$ for all $x \in A$. Let R be a nonempty relation on a set A. If R is symmetric and transitive, prove that R is not irreflexive.

(6 Marks)

(b) Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$ be a permutation of A.

i) Write p as a product of disjoint cycles.

ii) compute p^{-1}

iii) compute p^2

iv) Find the smallest positive integer k such that $p^k = 1_A$.

(8 Marks)

(c) If $(A, \leq)$ is a finite poset, prove that A has both a maximal and a minimal element.

(6 Marks)

6\. (a) In the following problems, consider the partial order of divisibility on the set A. Draw the Hasse diagram of the poset and determine whether the poset is linearly ordered (totally ordered) or not.

i) $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$

ii) $A = \{2, 4, 8, 16, 32\}$

(6 Marks)

(b) Consider the Hasse diagram of a poset given below. If $B = \{c, d, e\}$ find, if they exist

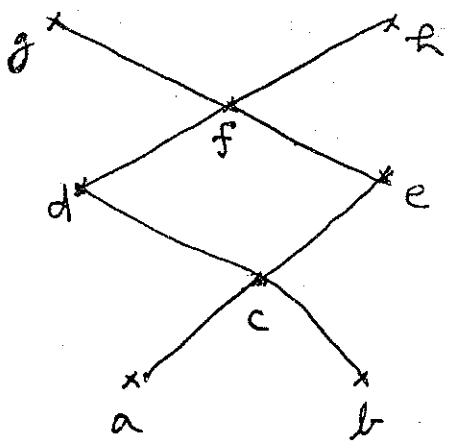
i) all upper bands of B

ii) all lower bands of B

iii) the least upper band of B

iv) the greatest lower band of B

(6 Marks)



Contd.... 4

(c) Let $A = \{1,2,3,4,5\}$. Define a relation R on $A \times A$ as $(x_1, y_1)R(x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$

i) Verify that R is an equivalence relation on A.

ii) Determine the equivalence classes $[(1,3)], [(2,4)]$ and $[(1,1)]$

iii) Determine the partition of A induced by R. **(8 Marks)**

7. (a) Let $R = \{s, t, x, y\}$. Define $+$ and $*$ making R into a ring by table i) for $+$ and by the partial table ii) for $*$

+	s	t	x	y
s	s	t	x	y
t	t	s	y	x
x	x	y	s	t
y	y	x	t	s

Table (i)

*	s	t	x	y
s	s	s	s	s
t	s	t	–	–
x	s	t	–	y
y	s	–	s	–

Table (ii)

Using associative and distributive laws, determine the entries for the missing spaces (denoted by –) in the table for *. **(5 Marks)**

(b) Let $G = \{q \in Q \mid q \neq -1\}$. Define a binary operation 0 on G as $x0y = x + y + xy$. Show that $(G, 0)$ is an abelian group. **(6 Marks)**

(c) State and prove Langranges Theorem. **(9 Marks)**

8. (a) For a group G, prove that the function $f : G \to G$ defined by $f(a) = a^{-1}$ is an isomorphism if and only if G is abelian. **(6 Marks)**

(b) Let $E : Z_2^m \to Z_2^m$ be an encoding function given by a generator matrix G or the associated parity - check matrix H. Prove that $C = E(Z_2^m)$ is a group code. **(8 Marks)**

(c) Prove that the set $(0,1) = \{x \in IR \mid 0 < x \leq 1\}$ is not a countable set. **(6 Marks)**

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CSI3

USN

Third Semester B.E. Degree Examination, July/August 2004

Computer/Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. All questions carry equal marks.

1. (a) Explain fundamental products with reference to three given sets A, B and C. (4 Marks)
   (b) Explain symmetric difference of two sets. (2 Marks)
   (c) Explain duality with four examples. (4 Marks)
   (d) Given $U = \{1, 2, 3....9\}$
   $A = \{1, 2, 5, 6\}$
   $B = \{2, 5, 7\}$
   $C = \{1, 3, 5, 7, 9\}$
   Find i) $A' \cap C'$
   ii) $A \oplus B$
   iii) $(B \oplus C) \setminus A$ (3+3+4=10 Marks)

2. (a) Given $R = \{(x, y) : x + 3y = 12\}$
   i) Write R as a set of ordered pairs.
   ii) Find domain, range, inverse of R.
   iii) Find $R \cdot R$. (2+4+2=8 Marks)
   (d) $A = \{a, b, c\}$ $R = \{(a, a)\ (a, b)\ (b, c)\ (c, c)\}$
   Find reflexive closure, symmetric closure and transitive closure. (12 Marks)

3. (a) What is Pigeonhole principle? (2 Marks)
   (b) Shirts numbered consecutively from 1 to 20 are worn by 20 students of a class. When any 3 of these students are chosen to be a team from the class, the sum of their shirt numbers is used as the code number of the team. Show that if any 8 of the 20 are selected then from these 8 we may form at least two different teams having the same code number. (10 Marks)
   (c) Prove that
   i) $r.nC_r = n.n-1C_{r-1}$
   ii) $_{n+1}C_r = nC_{r-1} +_n C_r$ (4+4=8 Marks)

4. (a) If a set A has n elements prove that its power set A has 2^n elements. (8 Marks)
   (b) Explain how explicit formula for $a_n = K1a_{n-1} + K2a_{n-2}\ \ n \geq 3$
   $a_1 = a$
   $a_2 = b$
   is obtained $K1, K2, a, b$ are constants. (8 Marks)
   (c) Prove by induction method.
   $\frac{1}{1.2} + \frac{1}{2.3} + ...\frac{1}{n(n+1)} = \frac{n}{n+1}$ (4 Marks)

Contd.... 2

5. (a) Solve the recurrence relation.

$a_n = 2a_{n/2} + n - 1 \quad n \geq 2 \quad a_1 = 0$

$n = 2^k$ where $k \geq 1$ (8 Marks)

(b) Given

$f(n) = 1000 + n - n^2 + logn$

$g(n) = n^{2.2}$

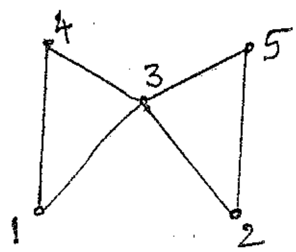
Prove that $f(n) = Og(n)$ (6 Marks)

(c) If R is a relation on the set of integers defined as $R = \{(a, b); 3 \text{ divides } a - b\}$ show that R is an equivalent relation. (6 Marks)

6. (a) Find explict formula for the Fibonacci sequence by formulating. (8 Marks)

(b) Draw the Hasse diagram for factors of 36. (8 Marks)

(c) Given Hasse diagram of a poset, find corresponding matrix. (4 Marks)



7. (a) Find the disfunctive normal form for $P \wedge (P \longrightarrow q)$ and find conjuctive normal form for $P \wedge (p \longrightarrow q)$ (8 Marks)

(b) Given $A = \{a, b\}$

*	a	b
a	b	a
b	a	b

(4 Marks)

Find whether A is a semigroup or a monoid. Justify your answer.

(c) Prove that in a boolean algebra for any a

i) $a \vee I = I$ ii) $a \wedge 0 = 0$ iii) $a \vee o = a$ iv) $a \wedge I = a$ (8 Marks)

8. Write short notes :

i) Tautology

ii) Warshall's Algorithm

iii) Lattices

iv) One to one and onto functions

v) Digraphs (5×4=20 Marks)

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NEW SCHEME

CS34

USN

Third Semester B.E. Degree Examination, July/August 2004

Computer Science /Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) Determine the sets A and B given that $A - B = \{1,2,4\}$, $B - A = \{7,8\}$ and $AUB = \{1,2,4,5,7,8,9\}$ (4 Marks)

(b) Using Venn diagrams, prove the following property of the symmetric difference :

$$A \triangle (B \triangle C) = (A \triangle B) \triangle C.$$ (6 Marks)

(c) Prove by mathematical induction :

$$1^2 + 3^2 + ... + (2n-1)^2 = \frac{n(2n-1)\,(2n+1)}{3}$$ (5 Marks)

(d) A survey on a sample of 25 new cars showed that the cars had the following options :

15 cars had air conditioners
12 cars had Radios
11 cars had power windows
5 cars had airconditioners and power windows
9 cars had air conditioners and radios
4 cars had radios and power windows
3 cars had all the three options

Find the number of cars that had

i) only power windows

ii) atleast one option (5 Marks)

2. (a) By constructing truth tables,

i) show that $[(p \vee q) \longrightarrow r] \Leftrightarrow [(p \longrightarrow r) \wedge (q \longrightarrow r)]$

ii) Examine whether $[(p \vee q) \longrightarrow r] \longleftrightarrow [\rceil r \longrightarrow \rceil (pvq)]$ is a tautology. (8 Marks)

(b) Define the dual of a logical statement. Write down the dual of

$$(P \vee T_0) \wedge (q \vee F_0) \vee (r \wedge\ s \wedge T_0)$$ (4 Marks)

(c) When is a conclusion p is said to follow from the premises $H_1, H_2, \ldots.., H_n$?

Let p, q, r be the primitive statements

p : Roger studies
q : Roger plays tennis
r : Roger passes in Discrete Mathematics

Let H_1, H_2 and H_3 be the premises

Contd.... 2

CSI32

Reg. No. [ ]

Third Semester B.E. Degree Examination, January/February 2003

Computer/Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) Given $U = \{1,2,3,4,5,6,7,8,9\}$
$A = \{1,2,4,6,8\}$
$B = \{2,4,5,9\}$ and $C = \{x|x \text{is a positive integer such that } x^2 \leq 16\}$,
Compute i) $\overline{A \cup B}$ ii) $B \oplus C$ (4 Marks)

(b) Prove that the set of all real numbers in (0, 1) is uncountable. (5 Marks)

(c) If f_A and f_B are characteristic functions of the sets A and B,
Prove that $f_{A \cap B} = f_A f_B$ (5 Marks)

(d) In a survey of 260 college students, the following data were obtained :
64 had taken mathematics course
94 had taken computer science course
58 had taken business course
24 had taken mathematics and business courses
26 had taken mathematics and computer science courses
22 had taken computer science and Business courses
14 had taken all the three courses.
Find the number of students who had taken
i) none of the courses
ii) only the computer science course. (6 Marks)

2. (a) State the rules for framing regular expressions. Show that
$a + b(ab)^*(a \times bva)$ is a regular expression on $A = \{+, x, ab\}$ (5 Marks)

(b) Prove by mathematical induction :
$1^3 + 2^3 + ...n^3 = \frac{n^2(n+1)^2}{4}$ (5 Marks)

(c) Find the number of ways in which a committee of 6 people can be selected from a group of 10 people if one of them is to be designated as the chairperson of the committee. (4 Marks)

(d) Solve the difference equation
$a_n = 3a_{n-1} - 2a_{n-2}$, given that $a_1 = 5$ and $a_2 = 3$ (6 Marks)

3. (a) Using rules of inference, show that Rvs follows from
$CVD, CVD \longrightarrow \urcorner H, \urcorner H \longrightarrow A \Lambda \urcorner B$ and $A \Lambda \urcorner B \longrightarrow RVS$. (7 Marks)

(b) Obtain the disjunctive normal form of $(\urcorner P \longrightarrow R) \Lambda (a \longleftrightarrow P)$ (7 Marks)

(c) Let A and B be two matrices of order n, Let $P(A, B) : A + B = I_n$.
Show that the truth value of $\forall A \exists BP(A, B)$ is T. (6 Marks)

4. (a) Let R and S be the relations, defined on $A = \{1,2,3\}$, whose matrices are

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad and \quad M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Obtain the matrices of the relations $\overline{R}, R^{-1}$ and $R \cap S$. (6 Marks)

Contd.... 2

(b) If R and S are equivalence relations on A, show that $R \cap S$ is also an equivalance relation on A. Give an example to show that $R \cup S$ is not an equivalence relation on A. (8 Marks)

(c) The relation $R = \{(a,b),(a,c),(b,b),(b,d),(b,e),(c,d),(d,e),(e,f)\}$ is defined on $A = \{a,b,c,d,e,f\}$. Obtain th relation R^2 without computing $M_R^{\ 2}$. (6 Marks)

5. (a) If $A = \{1,2,3,4\}$ and
$R = \{(1,1),(1,3),(3,2).(3,4),(4,2)\}$,
$S = \{(2,1),(3,3),(3,4),(4,1)\}$
Compute SoR, RoS.
Also find $|AXA - (RoR)|$ (6 Marks)

(b) Describe Warshall's algorithm for obtaining the transitive closure of a relation R starting from its adjacency matrix M_R.
Using Warshall's algorithm, Compute the matrix of the transitive closure of the relation whose adjacency matrix is

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$ (10 Marks)

(c) Obtain the equivalence relation R induced by the partition $\{\{1,2,3\},\{4,5\}\}$ of $\{1,2,3,4,5\}$ (4 Marks)

6. (a) Given $P_1 \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 4 & 6 \end{pmatrix}$ $P_2 \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{pmatrix}$

i) Compute $P_2 O P_1^{\ -1}$

ii) Express P_1 as a product of transpositions. (7 Marks)

(b) Show that if seven numbers are selected from 1 to 12, then two of them will add up to 13. (7 Marks)

(c) Explain the advantage of using Hashing functions for storing and accessing a large number of data records in a computer. (6 Marks)

7. (a) Define a Poset and explain how it can be represented in a Hasse diagram. Give an example. (7 Marks)

(b) Define a lattice. Let L be the power set of a set. Show that $(L, \subseteq)$ is a lattice where $\subseteq$ denotes the partial order "is a subset of ". (7 Marks)

(c) For any lattice $(L, \leq)$ prove :
$av(bvC) = (avb)vc$ (6 Marks)

8 (a) Define a semigroup and a monoid. Give an example each. (4 Marks)

(b) If $(G, *)$ is an abelian group, show, using mathematical induction, that
$(a * b)^n = a^n * b^n$. (5 Marks)

(c) Let $(S, *)$ and $(T, *')$ be two monoids with identities e and e' respectively. Let $f : S \longrightarrow T$ be an isomorphism from $(S, *)$ to $(T, *')$. Show that $f(e) = e'$. (6 Marks)

(d) If $G_1 = (A, *)$ and $G_2 = (B, *')$ are two groups, define the associated product group G_3, identifying the identity of G_3 and the inverses of elements of G_3. (5 Marks)

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CSI32

USN

Third Semester B.E. Degree Examination, July/August 2003

Computer/Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) Define :
   i) a power set
   ii) Partition of a set
   iii) Symmetric difference of two sets
   iv) Union of two sets each with an example. (8 Marks)

   (b) A computer company must hire 25 programmers to handle system programming jobs and 40 programmers for application programing. Of those hired, ten will be expected to perform jobs of both types. How many programmers must be hired? (6 Marks)

   (c) Prove by mathematical induction (6 Marks)

$$\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A_i}$$

2. (a) Explain the following, each with an example
   i) conjunction ii) disjunction iii) proposition. (6 Marks)

   (b) Prove that $n+1C_r = nC_{r-1} + nC_r$ (8 Marks)

   (c) Show that if seven numbers from 1 to 12 are chosen, then two of them add up to 13. (6 Marks)

3. (a) If R is a relation on the set of integers, defined as $R = \{(a,b) : 3 \text{ divides } (a-b)\}$, show that R is an equivalence relation. (8 Marks)

   (b) Define a directed graph, path and cycle. (4 Marks)

   (c) If $A = \{1,2,3,4,5\}$ and R is a relation on A where
   $R = \{(1,1)\ (1,2)\ (1,3)\ (1,5)\ (2,2)\ (2,3)\ (2,4)$
   $(3,3)\ (3,4)\ (3,5)\ (4,1)\ (4,2)\ (5,1)\ (5,2)\ (5,5)\}$
   Find the matrix of the relation M_R, also represent 'R' as a graph. (8 Marks)

4. (a) Define one-one and onto functions. If 'f' and 'g' are one-one and onto functions, show that g_{of} is a one-one and onto function. (8 Marks)

   (b) Define odd and even permutation, determine whether the following permutation is odd or even.
   $$P\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 2 & 1 & 6 & 5 & 8 & 7 & 3 \end{pmatrix}$$
   Where 'P' is a permutation on $A = \{1,2,3,4,5,6,7,8\}$ (6 Marks)

   (c) If R and S are two relations on the set A show that
   i) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$ ii) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$ (6 Marks)

Contd.... 2

5. (a) If 'R' and 'S' are equivalence relations, show that $(R \cup S)^{\infty}$ is smallest equivalence relation containing both R and S. (6 Marks)

   (b) Using Wonshall's algorithm, find the transitive closure of the relation whose matrix is

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(7 Marks)

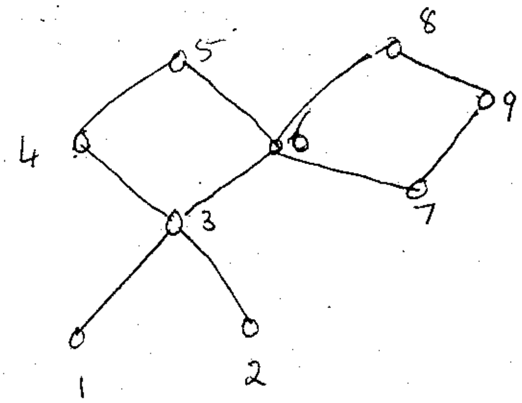
   (c) Define a Poset. Show that the relation 'R' defined on 'A' is a Poset where $A = \{P(s)\}$ and $S = \{a, b, c\}$ and 'R' is the relation of set inclusion. (7 Marks)

6. (a) If $(A, \leq)$ and $(B, \leq)$ are posets, then show that $(A \times B, \leq)$ is a Poset with partial order defined by $(a, b) \leq (a^1, b^1)$ if $a \leq a^1$ in A and $b \leq b^1$ in B. (8 Marks)

   (b) Find the Hasse diagram of $(A, \leq)$, where $A = D_{20}$ and $\leq = \{(a_1 b) : a/b\}$ (8 Marks)

   (c) Define upper bound, lower bound, GLB, LUB. (4 Marks)

7. (a) Determine LUB, GLB from the following Hasse diagram with respect to the sets $B_1 = \{3, 4, 6\}$ and $B_2 = \{4, 6, 9\}$ (8 Marks)



   (b) Define a Lattice, if 'L' is a lattice for every a, b in L prove that
   i) $a \cup b = b$ if and only if $a \leq b$
   ii) $a \cap b = a$ if and only if $a \leq b$ (8 Marks)

   (c) Define a semi group and a group. (4 Marks)

8. (a) If in a group, every element is its own inverse. show that the group is abelian. (7 Marks)

   (b) If $(S, *)$ and $(T, *^1)$ are semi groups, show that $S \times T, *^{11})$ is a semi group. Where $*^{11}$ is defined as $(S_1, t_1) *^{11} (s_2, t_2) = (s_1 * s_2, t_1 *^1 t_2)$. (6 Marks)

   (c) Let 'G' be a group show that the function $f : G \to G$ defined by $f(a) = a^2$, is a homomorphism if and only if 'G' is abelian. (7 Marks)

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(Pages : 3)

CSI—32

THIRD SEMESTER B.E. (COMPUTER SCIENCE AND INFORMATION SCIENCE ENGINEERING) DEGREE EXAMINATION, AUGUST/SEPTEMBER 2000

DISCRETE MATHEMATICAL STRUCTURE

Time : Three Hours

Maximum : 100 Marks

Answer any **five** *questions.*
All questions carry equal marks.

I. (a) Define Power set, symmetric difference of two sets and complement of a set. Give an example for each.

(6 marks)

(b) Show that for any *two* sets A and B :

$A - B = A - (A \cap B)$.

(6 marks)

(c) A survey on a sample of 25 new cars being sold at a auto dealer was conducted to see which of the three popular options, air-conditioning (A), radio (R), and power windows (W) were already installed. The survey found :

15	had	air-conditioning
12	had	radio
11	had	power windows
5	had	air-conditioning and power windows
9	had	air-conditioning and radio
4	had	radio and power windows
3	had	all three options

Find the number of cars that had :

(i) Only power windows. (ii) Only air-conditioning.
(iii) Only radio. (iv) Only one of the option.
(v) At least one option. (vi) None of the options.

(8 marks)

II. (a) Find the regular sets of the three regular expressions shown below with $I = \{0, 1\}$:

(i) $0 * (0 \vee 1) *$; (ii) $00* (0 \vee 1) * 1$; (iii) $(01)* (01 \vee 1*)$.

(6 marks)

(b) Prove by mathematical induction $n! \geq 2^{n-1}$.

(6 marks)

(c) A woman has 11 close friends :

(i) In how many ways can she invite five of them to dinner ?

(ii) In how many ways, if two of the friends married, and will not attend separately ?

(iii) In how many ways if two of them are not on speaking terms and will not attend together ?

(8 marks)

Turn over

III. (a) Show that truth values of the following statements independent of thei component :

(i) $(P \wedge (P \rightarrow Q)) \rightarrow Q$.

(ii) $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$.

(6 marks

(b) Find the disjunctive normal form of $\neg (P \vee Q) \leftrightarrow (P \wedge Q)$. (6 marks

(c) Show that SVR is tautologically implied by $(P \vee Q)$, $(P \rightarrow R)$ and $(Q \rightarrow S)$.

(8 marks

IV. (a) Prove that for any two non-empty sets A and B :

$|A \times B| = |A|\ |B|$.

(6 marks

(b) Define Partition of a set with an example. (4 marks

(c) Let A = B = {1, 2, 3, 4, 6} ; aRb if and only if a is multiple of b. Find the domain range represent the relation as matrix and draw the digraph of the relation.

(6 marks

(d) Let R = {<1, 2>, <3, 4>, <2, 2>} and S = {<4, 2>, <2, 5>, <3, 1>, <1, 3>}. Find ROS RO (SOR), RO ROR, SOS.

(4 marks

V. (a) Determine whether the relation is reflexive, symmetric, antisymmetric o transitive :

A = {1, 2, 3, 4}, R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)}.

(4 marks

(b) Briefly discuss the methods involved in representation of relation and digraph i computer.

(10 marks

(c) Let A = {a, b, c, d, e} and R = {(a, a), (a, b), (b, c), (c, e), (c, d), (d, e)}. Compute (i) R^2 ; (ii) R^∞.

(6 marks

VI. (a) Let X = {1, 2, 3, 4}. Determine whether or not each relation below from X into X i a function :

(i) f = {(2, 3), (1, 4), (2, 1), (3, 2), (4, 4)}.

(ii) g = {(3, 1), (4, 2), (1, 1)}.

(iii) h = {(2, 1), (3, 4), (1, 4), (2, 1), (4, 4)}.

(6 marks

(b) Let R be the set of real numbers and let $f : R \rightarrow R$ defined by $f(x) = x^2$, is f i invertible.

(6 marks

(c) Show that if 30 dictionaries in a library contain of 61,327 pages, then one of th dictionaries must have at least 2045 pages. (4 marks

(d) Let A = {1, 2, 3}. Find all permutations of A. Compute (i) inverse of any one of the permutation ; (ii) product of two permutations.

(6 marks)

VII. (a) Define Poset with an example. (4 marks)

(b) Draw the Hasse diagram of divisor of 36. (4 marks)

(c) For any lattice, prove that :

(i) $a \vee (b \vee c) = (a \vee b) \vee c$.

(ii) $a \vee (a \wedge b) = a$.

(8 marks)

(d) Define Boolean algebra with an example. (4 marks)

VIII. (a) Define Isomorphism, homomorphism and cyclic group with respect to group/semigroup with an example.

(6 marks)

(b) Let Z be the set of integers and T be the set of all even integers. Show that semigroups (Z, +) and (T, +) are isomorphic.

(6 marks)

(c) Let G be the set of non-zero real numbers and let $a * b = ab/2$. Show that (G, $*$) is an abelian group.

(8 marks)