10MAT41

## Fourth Semester B.E. Degree Examination, June/July 2013 Engineering Mathematics - IV

Time: 3 hrs .
Max. Marks: 100

## Note: 1. Answer FIVE full questions, selecting at least TWO questions from each part. <br> 2. Use of Statistical tables permitted.

## PART - A

1 a. Use modified Euler's method to solve $\frac{d y}{d x}=x+y, y(0)=1$ at $x=0.1$ for three iterations taking $\mathrm{h}=0.1$.
(06 Marks)
b. Solve $\frac{d y}{d x}=x+y, x=0, y=1$ at $x=0.2$ using Runge-Kutta method. Take $h=0.2$.
(07 Marks)
๑. Using Milne's predictor-corrector method find $y(0.3)$ correct to three decimals given,
(07 Marks)

| x | -0.1 | 0 | 0.1 | 0.2 |
| :---: | :---: | :---: | :---: | :---: |
| y | 0.908783 | 1.0000 | 1.11145 | 1.25253 |

2 a. Approximate $y$ and $z$ at $x=0.2$ using Picard's method for the solution of $\frac{d y}{d x}=z$, $\frac{d z}{d x}=x^{3}(y+z)$ with $y(0)=1, z(0)=1 / 2$. Perform two steps $\left(y_{1}, y_{2}, z_{1}, z_{2}\right)$.

1. Using Runge-Kutta method solve $\mathrm{y}^{\prime \prime}=\mathrm{x}\left(\mathrm{y}^{\prime}\right)^{2}-\mathrm{y}^{2}$ at $\mathrm{x}=0.2$ with $\mathrm{x}_{0}=0, \mathrm{y}_{0}=1, \mathrm{z}_{0}=0$ take $\mathrm{h}=0.2$.
(10 Marks)
3 a. If $f(z)=u+i v$ is analytic prove that Cauchy-Reimann equations $u_{x}=v_{y}, u_{y}=-v_{x}$ are true.
b. If $w=z^{3}$ find $d w / d z$.
(06 Marks)
๔. If the potential function is $\phi=\log \sqrt{\mathrm{x}^{2}+\mathrm{y}^{2}}$. Find the stream function.
(07 Marks)
(07 Marks)

4 a. Find the bilinear transformation which maps the points $\mathrm{z}=1, \mathrm{i},-1$ onto the points $\mathrm{w}=\mathrm{j}, \mathrm{o},-\mathrm{i}$.
(06 Marks)
b. Discuss the conformal transformation $w=e^{z}$. Any horizontal strip of height $2 \pi$ in z-plane will map what portion of w-plane.
(07 Marks)
c. State and prove Cauchy's integral formula.
(07 Marks)
PART - B
5 a. Prove that $J_{1 / 2}^{(x)}=\sqrt{\frac{2}{\pi x}} \sin x$.
(06 Marks)
b. State and prove Rodrigues formula for Legendre's polynomials.
(07 Marks)
c. Express $f(x)=x^{4}+3 x^{3}-x^{2}+5 x-2$ in terms of Legendre polynomial.
(07 Marks)

6 a. The probabilities of four persons A, B, C, D hitting targets are respectively $1 / 2,1 / 3,1 / 4,1 / 5$. What is the probability that target is hit by atleast one person if all hit simultaneously?
(06 Marks)
b. i) State addition law of probability for any two events A and B.
ii) Two different digits from 1 to 9 are selected. What is the probability that the sum of the two selected digits is odd if ' 2 ' one of the digits selected.
(07 Marks)
c. Three machine A, B, C produce $50 \%, 30 \%, 20 \%$ of the items. The percentage of defective items are $3,4,5$ respectively. If the item selected is defective what is the probability that it is Crom machine A? Also find the total probability that an item is defective.
(07 Marks)
7 a. The p.d.f of $x$ is

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{p}(\mathrm{x})$ | k | 3 k | 5 k | 7 k | 9 k | 11 k | 13 k |

Find $k$. Also find $\mathrm{p}(\mathrm{x} \geq 5), \mathrm{p}(3<\mathrm{x} \leq 6)$.
(06 Marks)
b. A die is thrown 8 times. Find the probability that ' 3 ' falls,
i) Exactly 2 times
ii) At least once
iii) At the most 7 times.
(07 Marks)
c. In a certain town the duration of shower has mean 5 minutes. What is the probability that shower will last for i) 10 minutes or more; ii) less than 10 minutes; iii) between 10 and 12 minutes.
(07 Marks)
8 a. What is nuil hypothesis, alternative hypothesis significance level?
(06 Marks)
b. The nine items of a sample have the following values: $45,47,50,52,48,47,49,53,51$. Does the mean of these differ significantly from the assumed mean of 47.5 . Apply student's t -distribution at $5 \%$ level of significance. ( $\mathrm{t}_{0.05}$ for $8 \mathrm{df}=2.31$ ).
(07 Marks)
c. In experiments on a pea breading, the following frequencies of seeds were obtained:

| Round-yellow | Wrinkled yellow | Round green | Wrinkled green | Total |
| :---: | :---: | :---: | :---: | :---: |
| 315 | 101 | 108 | 32 | 556 |

Is the experiment is in the agreement of theory which predicts proportion of frequencies 9:3:3:1 ( $\mathrm{x}_{0.05}^{2}, 3 \mathrm{df} \equiv 7.815$ ).
(07 Marks)

