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10MAT31

Third Semester B.E. Degree Examination, December 2012
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Find the Fourier series of $f(x) = x - x^2$, $-\pi \le x \le \pi$. Hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots\ldots\ldots = \frac{\pi^2}{12}$$ **(07 Marks)**

Is the above deduced series convergent? (Answer in Yes or No)

b. Define : i) Half range Fourier sine series of f(x)
ii) Complex form of Fourier series of f(x)

Find the half range cosine series of $f(x) = x$ in $0 < x < 2$. **(07 Marks)**

c. Obtain a_0, a_1, b_1 in the Fourier expansion of y, using harmonic analysis for the data given.

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(06 Marks)

2 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

Hence evaluate $\displaystyle\int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos\left(\frac{x}{2}\right)dx$ 5 - 6:30 **(07 Marks)**

b. Find the Fourier sine transform of $\dfrac{e^{-ax}}{x}$ **(07 Marks)**

c. Find the Fourier cosine transform of

$$f(x) = \begin{cases} 4x, & \text{for } 0 < x < 1 \\ 4 - x, & \text{for } 1 < x < 4 \\ 0, & \text{for } x > 4 \end{cases}$$ **(06 Marks)**

3 a. i) Write down the two dimensional heat flow equation (p d e) in steady state (or two dimensional) Laplace's equation. Just mention.

ii) Solve one dimensional heat equation by the method of separation of variables. **(07 Marks)**

b. Using D'Alembert's method, solve one dimensional wave equation. **(07 Marks)**

c. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form of $y = a \sin(\pi x/l)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is,

$$y(x,t) = a\sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi ct}{\ell}\right)$$

Start the answer assuming the solution to be

$$y = (C_1\cos(px) + C_2\sin(px))(C_3\cos(cpt) + C_4\sin(cpt))$$ **(06 Marks)**

4 a. Fit a linear law, $P = mW + C$, using the data

P	12	15	21	25
W	50	70	100	120

(06 Marks)

b. Find the best values of a and b by fitting the law $V = at^b$ using method of least squares for the data,

V (ft/min)	350	400	500	600
t (min)	61	26	7	26

Use base 10 for algorithm for computation. (07 Marks)

c. Using simplex method,
Maximize $Z = 5x_1 + 3x_2$
Subject to, $x_1 + x_2 \le 2$; $5x_1 + 2x_2 \le 10$; $3x_1 + 8x_2 \le 12$; $x_1, x_2 \ge 0$. (07 Marks)

PART – B

5 a. Use Newton-Raphson method, to find the real root of the equation $3x = (\cos x) + 1$.
Take $x_0 = 0.6$. Perform two iterations. (06 Marks)

b. Apply Gauss-Seidel iteration method to solve equations

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3z + 20z = 25$$

Assume initial approximation to be $x = y = z = 0$. Perform three iterations. (07 Marks)

c. Using Rayleigh's power method to find the largest eigen value and the corresponding eigen vector of the matrix.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Take $[1 \ 0 \ 0]^T$ as the initial approximation. Perform four iterations. (07 Marks)

6 a. Use appropriate interpolating formula to compute y(82) and y(98) for the data

x	80	85	90	95	100
y	5026	5674	6362	7088	7854

(07 Marks)

b. i) For the points (x_0, y_0) (x_1, y_1) (x_2, y_2) mention Lagrage's interpolation formula.
ii) If $f(1) = 4$, $f(3) = 32$, $f(4) = 55$, $f(6) = 119$; find interpolating polynomial by Newton's divided difference formula. (07 Marks)

c. Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$, using

i) Simpson's $1/3^{rd}$ rule ii) Simpson's $3/8^{th}$ rule iii) Weddele's rule, using

x	0	1	2	3	4	5	6
$f(x) = \frac{1}{1+x^2}$	1	0.5	0.2	0.4	0.0588	0.0385	0.027

(06 Marks)

7 a. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to u(0, t), u(4, t) = 0. $u_t(x, 0) = 0$ and u(x, 0) = x(4 - x) by taking h = 1, k = 0.5 upto four steps. **(07 Marks)**

b. Solve two dimensional Laplace equation at the pivotal or nodal points of the mesh shown in Fig.Q7(b). To find the initial values assume $u_4 = 0$. Perform three iterations including computation of initial values. **(07 Marks)**

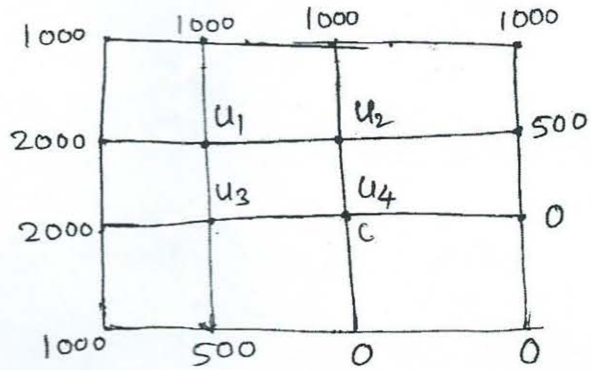

Fig.Q7(b)

c. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the conditions u(x, o) = $\sin\pi x$, $o \le x \le 1$; u(0, t) = u(1, t) = 0. Carry out computations for two levels, taking h = 1/3 , k = 1/36. **(06 Marks)**

8 a. Find the z-transform of

$$\frac{n}{3^n} + 2^n n^2 + 4\cos(n\theta) + 4^n + 8$$ **(07 Marks)**

b. State and prove i) Initial value theorem ii) Final value theorem of z-transforms. **(07 Marks)**

c. Using the z-transform solve

$u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0$, $u_1 = 1$. **(06 Marks)**

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