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06MAT41

Fourth Semester B.E. Degree Examination, December 2010

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting at least TWO questions from each part.***
2. Any missing data may be suitably assumed.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Given $\frac{dy}{dx}+y-x^2=0$, $y(0)=1$, $y(0.1)=0.9052$, $y(0.2)=0.8213$. Find correct to four decimal places y(0.3) and y(0.4) using modified Euler's method. **(07 Marks)**

b. Apply Runge – Kutta method of order four, to compute y(2.0). Given $10\frac{dy}{dx}=x^2+y^2$, $y(0)=1$, taking $h=0.1$. **(07 Marks)**

c. The following table gives the solution of $\frac{dy}{dx}=x-y^2$. Find the value of y at x = 0.8, using Milne's predictor and corrector formulae.

X	0	0.2	0.4	0.6
Y	0	0.02	0.07	0.17

(06 Marks)

2 a. Show that polar forms of Cauchy's Riemann equation are $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial V}{\partial \theta}, \frac{\partial V}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$. Deduce that $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$. **(07 Marks)**

b. Determine the analytic function $w = u + i\upsilon$ if $V = \log(x^2+y^2)+x-2y$. **(07 Marks)**

c. Find the Bilinear transformation which maps the points z = 1, i, -1 into w = 0, 1, ∞. **(06 Marks)**

3 a. State and prove Cauchy's integral formula. **(07 Marks)**

b. Find the Laurent series of $\frac{3x^2-6z+2}{z^3-3z^2+2z}$. i) $1<|z|<2$ ii) $|z|>2$. **(07 Marks)**

c. Evaluate $\int_c \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)^2(z-2)}dz$, where c is $|z|=3$ using Cauchy's residue theorem. **(06 Marks)**

4 a. Solve the equation in series $\frac{d^2y}{dx^2}+x^2y=0$. **(07 Marks)**

b. Obtain the series solution of Bessel's differential equation in the form $y=AJ_n(x)+BJ_{-n}(x)$. **(07 Marks)**

c. If $x^3+2x^2-x+1=aP_0(x)+bP_1(x)+cP_2(x)+dP_3(x)$, find the value of a, b, c, d. **(06 Marks)**

PART – B

5 a. Fit a curve of form $y = ab^x$ and hence estimate y when x = 8.

X	1	2	3	4	5	6	7
Y	87	97	113	129	202	195	193

(07 Marks)

b. If θ is the angle between the lines of regression then show that

$$\tan\theta = \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\left(\frac{1-r^2}{r}\right)$$

(07 Marks)

c. State and prove Baye's theorem. **(06 Marks)**

6 a. The pdf of a variate X is given by the following table :

X	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

For what value of k, this represents a valid probability distribution?
Also find : i) $P(x \geq 5)$ ii) $P(3 < x \leq 6)$. **(07 Marks)**

b. Given that 2% of the fuses manufactured by a firm are defective, find by using Poisson distribution, the probability that a box containing 200 fuses has
i) No defective fuses ii) 3 or more defective fuses iii) At least one defective fuse. **(07 Marks)**

c. The marks of 100 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be i) less than 65 ii) more than 75 iii) between 65 and 75. **(06 Marks)**

7 a. Explain the following terms :
i) Null hypothesis ii) Type I and type II error iii) Confidence limits. **(07 Marks)**

b. A sample of 100 days is taken from a coastal town of a certain district and of 10 of them are found to be very hot. What are the probable limits of the percentage of hot days in the district? **(07 Marks)**

c. A certain stimulus administrated to each of the 12 patients resulted in the following change in blood pressure.
5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? ($t_{0.05}$ for 11 df = 2.201). **(06 Marks)**

8 a. The joint probability distribution of two random variables x and y is as follows :

x \ y	-2	-1	4	6
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine :
i) The marginal distribution of x and y ii) Co variance of x and y iii) Correlation of x and y. **(07 Marks)**

b. Verify that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 \end{bmatrix}.$$

is a regular stochastic matrix. **(07 Marks)**

c. Explain:
i) Absorbing state of Markov chain ii) Transient state iii) Recurrent state. **(06 Marks)**

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