NEW SCHEME

EE52

USN

Fifth Semester B.E. Degree Examination, January/February 2005

Electrical & Electronics Engineering

Modern Control Theory

Time: 3 hrs.] [Max.Marks : 100

Note: ***Answer any FIVE full questions.***

1. (a) Mention the disadvantages of conventional control theory and explain how these are overcome in modern control theory with particular reference to (i) nonlinear systems (ii) time varying system (iii) Analysis (iv) design and (v) computer applications. **(10 Marks)**

(b) Obtain the state space representation in phase variable canonical form for the system represented by

$D^4y+20D^3y+45D^2y+18Dy+100y=10D^2u+5Du+100u$ with y as output and u as input. **(10 Marks)**

2. (a) Determine the transfer matrix for the system **(10 Marks)**

$$\begin{bmatrix}\mathring{x}_1\\ \mathring{x}_2\end{bmatrix}=\begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}4 & 6\\ -5 & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}:\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}1 & -1\\ 8 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

(b) For a system represented by $\mathring{x}=Ax$ the response to one set of initial conditions is $x(t)=\begin{bmatrix}2e^{-4t}\\ e^{-4t}\end{bmatrix}$ and another set of initial condition is $x(t)=\begin{bmatrix}4e^{-2t}\\ e^{-2t}\end{bmatrix}$

Determine the matrix A and the state transition matrix $\phi(t)$ **(10 Marks)**

3. (a) Mention the conditions for complete controllability and complete observability of continuous time systems. Using these, explain the principle of duality between controllability and observability. **(10 Marks)**

(b) Use controllability and observability matrices to determine whether the system represented by the flow graph shown in Fig 1. is completely controllable and completely observable **(10 Marks)**

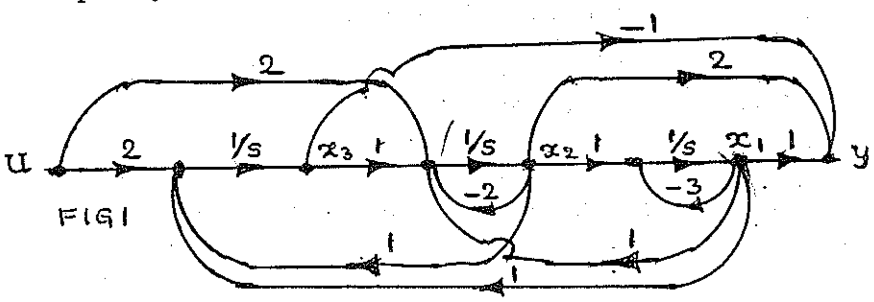

FIG 1

4. (a) Given the time invariant system

$$\begin{bmatrix} \overset{\circ}{x}_1 \\ \overset{\circ}{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \alpha \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u : y = [1\ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and that $u(t) = e^{-1}$ and $y(t) = 2 - \alpha t e^{-1}$, find $x_1(t)$ and $x_2(t)$. Find also $x_1(0)$ and $x_2(0)$. What happens if $\alpha = 0$? **(10 Marks)**

(b) Find the transformation matrix P that transforms the matrix A into diagonal or Jordon form, where

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 1 & 0 & 2 \\ 1 & -1 & 3 \end{bmatrix}$$

(10 Marks)

5. (a) Explain the following behaviour of nonlinear systems

i) Frequency amplitude dependence

ii) Multivalued responses and jump resonances. **(10 Marks)**

(b) Determine the kind of singularity for each of the following differential equations.

i) $\ddot{y} + 3\dot{y} + 2y = 0$

ii) $\ddot{y} - 8\dot{y} + 17y = 34.$ **(10 Marks)**

6. (a) What is a phase plane plot? Describe delta method or any other method of drawing phase plane trajectories. **(10 Marks)**

(b) Draw the phase plane trajectory for the system described by the differential equation.

$D^2x + x = 0$ with initial conditions $x(o) = 1$ and $Dx(o) = 0$. **(10 Marks)**

7. (a) Prove that the necessary and sufficient condition for arbitrary pole placement in that system be completely state controllable. **(10 Marks)**

(b) An observable system is described by

$$\overset{\circ}{x} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} u \; ; \; y = [0\ 0\ 1]x$$

Design a state observer so that the eigen values are at -4 ; $-3 \pm J1$. **(10 Marks)**

8. (a) State and explain Liapunov's theorems on

i) asymptotic stability

ii) global aeymplotic stability and

iii) instability. **(10 Marks)**

(b) Use Krasovskil's theorem to show that the equilibrium state $x = 0$ of the system described by

$$\overset{\circ}{x}_1 = -3x_1 + x_2$$

$$\overset{\circ}{x}_2 = x_1 - x_2 - x_2^3$$

is asymptotically stable in the large. **(10 Marks)**

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NEW SCHEME

USN | | | | | | | | | | |

Fifth Semester B.E. Degree Examination, July/August 2005

Electrical & Electronics Engineering

Modern Control Theory

Time: 3 hrs.] [Max.Marks : 100

Note: ***Answer any FIVE full questions.***

1. (a) Explain effects of a PI controller on the static and dynamic response of a system. **(5 Marks)**

(b) Consider a typical second order, type one system with unity feedback, being controlled by a PD controller and show that

i) Damping increases with PD control

ii) Steady state error to a ramp input remains unchanged if proportional gain $k_p = 1$. **(8 Marks)**

(c) Obtain the state model of the electrical network shown in fig.1(c) selecting $v_1(t)$ and $v_2(t)$ as state variables. **(7 Marks)**

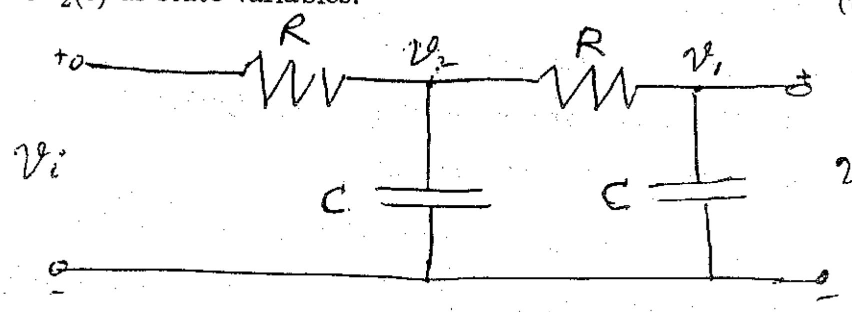

Fig. 1(c)

2. (a) Obtain the state model of the system represonsed by the following differential equation

$\dddot{y} + 6\ddot{y} + 5\dot{y} + y = u.$ **(5 Marks)**

(b) Obtain two different state models for a system represented by the following transfer function. Write suitable state diagram in each case.

$\frac{y(s)}{u(s)} = \frac{8s^2+17s+8}{(s+1)\ (s^2+8s+15)}$ **(8 Marks)**

(c) Obtain the state model of the system represented by the following transfer function in Jordan canonical form. Write the state diagram.

$\frac{y(s)}{u(s)} = \frac{2s^2+6s+5}{(s^2+2s+1)\ (s+2)}$ **(7 Marks)**

3. (a) What are generalised eigen vectors? How are they determined? (5 Marks)

(b) Convert the following state model into canonical form

$$\overset{\circ}{x} = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u$$
$$y = [1 \;\; 0]x.$$

(8 Marks)

(c) Convert the following square matrix A into Jordan canonical form using a suitable non singular transformation matrix P.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -9 & -6 \end{bmatrix}$$

(7 Marks)

4. (a) What is a state transition matrix? List the properties of state transition matrix. (6 Marks)

(b) Given the state model of a system.

$$\overset{\circ}{x} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = [1 \;\; 0]x.$$

with initial conditions $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Determine :

i) The state transition matrix

ii) The state transition equation $x(t)$ and output $y(t)$ for an unit step input

iii) Inverse state transition matrix. (14 Marks)

5. (a) Explain the concept of controllability and observability. (6 Marks)

(b) Determine the controllability and observability of the following state model

$$\overset{\circ}{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = [10 \;\; 5 \;\; 1]x$$

(8 Marks)

(c) A system represented by following state model is controllable but not observable. Show that the non-observability is due to a pole-zero cancellation in $C[sI - A]^{-1}$. (6 Marks)

$$\overset{\circ}{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = [1 \;\; 1 \;\; 0]x$$

6. (a) Write the block diagram of a system with observer based state feedback controller. (5 Marks)

(b) It is desired to place the closed loop poles of the following system at $s = -3$ and $s = -4$ by a state feedback controller with the control law $u = -Kx$. Determine the state feedback gain matrix K and the control signal.

$$\mathring{x} = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$
$$y = [1 \quad 0]x$$

(7 Marks)

(c) Consider the system represented by

$$\mathring{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = [1 \quad 0 \quad 0]x$$

Design a full order observer such that the observer eigen values are at $-2 \pm \rho 2\sqrt{3}$ and -5.

(8 Marks)

7. (a) With reference to non-linear system explain :

i) Jump resonance ii) Limit cycles.

(6 Marks)

(b) What are singular points? Explain the classification of singular points based on the location of eigen values of the system.

(8 Marks)

(c) Explain the construction of a phase trajectory either by isocline method or by delta method.

(6 Marks)

8. (a) Define :

i) Stability

ii) Asymptotic stability

iii) Asymptotic stability in the large.

(5 Marks)

(b) Investigate the stability of the following nonlinear system using direct method of Liapunov.

$$\mathring{x}_1 = x_2$$
$$\mathring{x}_2 = -x_1 - x_1^2 x_2$$

(5 Marks)

(c) A second order system is represented by

$\mathring{x} = Ax$ where $A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$.

Assuming matrix Q to be identity matrix, solve for matrix P in the equation $A^T P + PA = -Q$. Use Liapunov theorem and determine the stability of the origin of the system. Write the Liapunov function $V(x)$.

(10 Marks)

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NEW SCHEME

EE52

Reg. No. [ ]

Fifth Semester B.E. Degree Examination, January/February 2006

Electrical & Electronics Engineering

Modern Control Theory

Time: 3 hrs.)

(Max.Marks : 100

Note: ***Answer any FIVE full questions.***

1. (a) Explain the concepts of state variable, state and state model of a linear system. **(6 Marks)**

(b) Linearize the following equation in the neighbourhood of the origin

$$\frac{d^2\theta}{dt^2} = tan^{-1} 2\theta - 3 \sin \theta + 2ue^{u/2} + u^3$$

Obtain the approximate response $\theta(t)$ for $u = 0.02$, with the system initially at equilibrium. **(6 Marks)**

(c) Choosing appropriate physical variables as state variables, obtain the state model for the electric circuit shown in Fig. 1. **(8 Marks)**

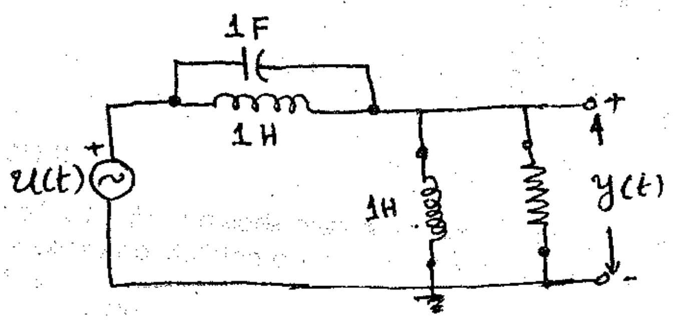

Fig. 1

2. (a) For the transfer function $\frac{Y(s)}{R(s)} = \frac{s(s+2)(s+3)}{(s+1)^2\ (s+4)}$

Obtain the state model in

i) Phase variable cannonical form

ii) Jordan Cannonical form

(4+4=8 Marks)

(b) Consider the matrix

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

i) Find the eigen values and eigen vectors of A

ii) Write the modal matrix

iii) Show that the modal matrix indeed diagonalizes A. **(12 Marks)**

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3. **(a)** List at least three important properties of the state transition matrix. **(3 Marks)**

(b) Consider the homogeneous equation $\dot{X} = AX$. Where A is a 3×3 matrix. The following three solutions for three different initial conditions are available

$$\begin{bmatrix} e^{-t} \\ -e^{-t} \\ 2e^{-t} \end{bmatrix}, \quad \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2e^{-3t} \\ -6e^{-3t} \\ 0 \end{bmatrix}$$

i) Identify the initial conditions

ii) Find the state transition matrix

iii) Hence or otherwise find the system matrix A **(10 Marks)**

(c) Given the state model $\dot{X} = AX + bu, \ y = cX$

Where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $c = [1 \ 0 \ 0]$

i) Simulate and find the transfer function $\frac{Y(s)}{U(s)}$ using Mason's gain formula.

ii) Determine the transfer function from the state model formulation. **(7 Marks)**

4. **(a)** Obtain the time response $y(t)$, of the system given below by first transforming the state model into a 'Cannonical model'.

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u, \ y = [1 \ 0 \ 0]x$$

u is a unit step function and $X^T(0) = [0 \ 0 \ 2]$ **(12 Marks)**

(b) Write the state and output equations for the system shown in Fig. 2. Determine whether the system is completely controllable and completely observable. **(8 Marks)**

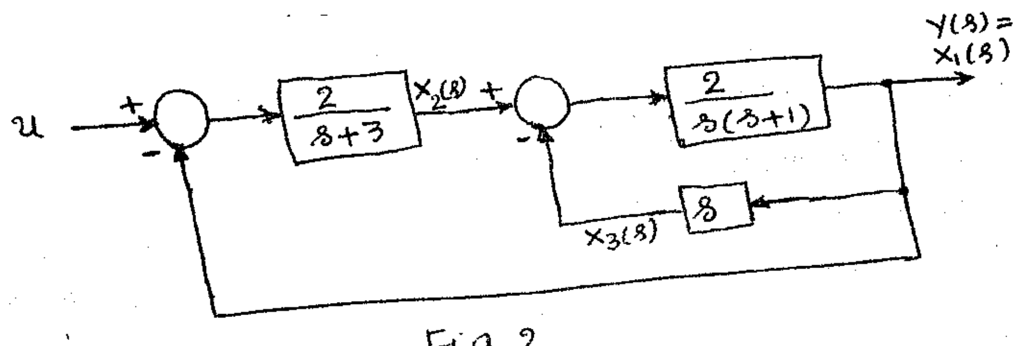


Fig 2.

5. A regulator system has the plant

$$\dot{X} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u, \ y = [0 \ 0 \ 1]X$$

i) Compute K so that the control law $u = -KX + r(t)$, $r(t)$ = reference input, places the closed loop poles at $-2 \pm j\sqrt{12}, \ -5$. **(8 Marks)**

ii) Design an observer such that the eigen values of the observer are located at $-2 \pm j\sqrt{12}, \ -5.$ **(6 Marks)**

iii) Draw a block diagram implementation of the control configuration. **(3 Marks)**

iv) Obtain the state model of the observer based state feed back control system. **(3 Marks)**

6. (a) Using Lyapunovs direct method, find the range of K to guarantee stability of the system shown in Fig. 3 **(14 Marks)**

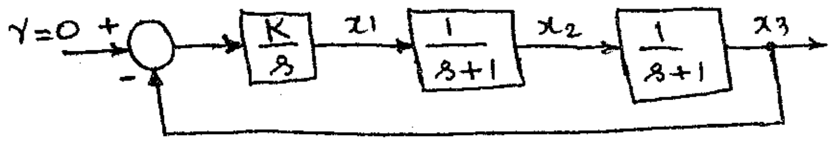


Fig 3

(b) Choose an appropriate Lyapunov function and check the stability of the equilibrium state of the system described by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - x_1^2\, x_2$$
(6 Marks)

7. (a) Discuss the basic features of the following non linearities

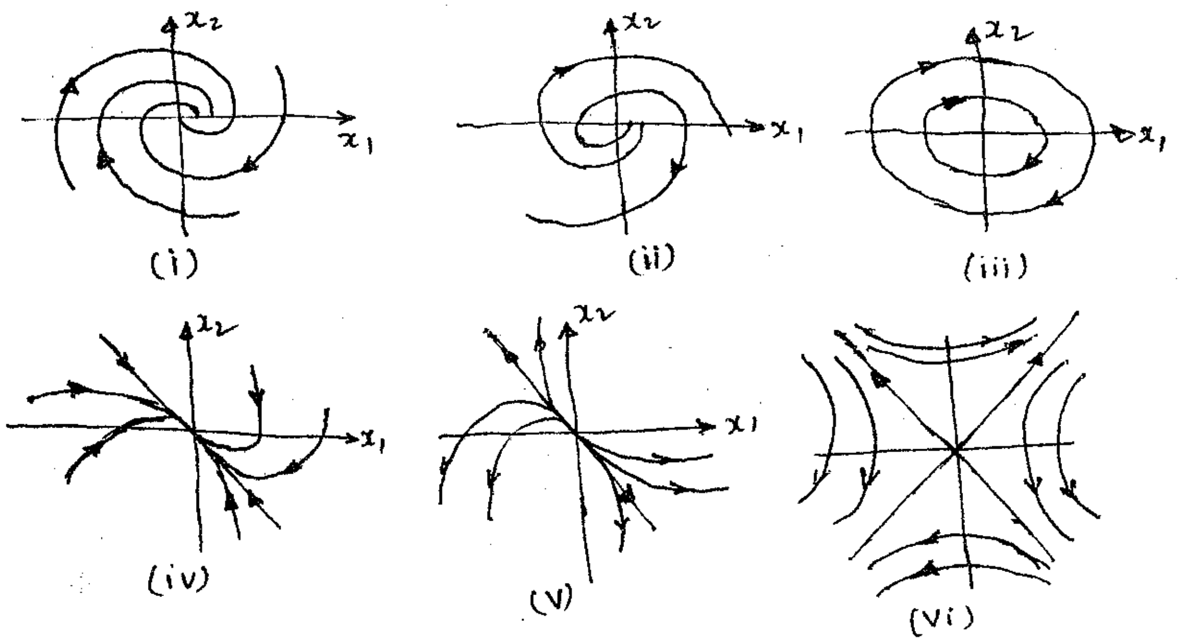
i) Non linear friction

ii) On-off controllers

iii) Back lash

(9 Marks)

(b) Fig. 4 shows phase portraits for type - 0 systems. Classify them into the categories. Stable focus, stable node and so-on. **(6 Marks)**



(c) Explain the concept of jump resonance with a suitable example. **(5 Marks)**

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8. **(a)** Explain the delta method of constructing phase trajectories. **(7 Marks)**

(b) Using Isocline method, draw the phase trajectory for the system

$$\frac{d^2x}{dt^2} + 0.6\frac{dx}{dt} + x = 0$$

with $x = 1$ and $\frac{dx}{dt} = 0$ as initial condition. **(8 Marks)**

(c) Sketch a suitable phase trajectory and explain the 'Dither phenomenon'. **(5 Marks)**

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