UNIT - 1 INTRODUCTION: When simulation is the appropriate tool and when it is not appropriate; Advantages and disadvantages of Simulation; Areas of application; Systems and system environment; Components of a system; Discrete and continuous systems; Model of a system; Types of Models; Discrete-Event System Simulation; Steps in a Simulation Study. Simulation examples: Simulation of queuing systems; Simulation of inventory systems; other examples of simulation. 8 Hours

What is Simulation?

A Simulation is the imitation of the operation of a real-world process or system over time.

- It can be done by hand or on a computer.
- The behavior of a system as it evolves over time is studied by developing a simulation model.
- This model takes the form of a set of assumptions concerning the operation of the system.
- The assumptions are **expressed** in
    1. Mathematical relationships
    2. Logical relationships
    3. Symbolic relationships between the entities of the system.

Why Simulation?

- Accurate Depiction of Reality
- Insightful system evaluations

1.1 When Simulation is the Appropriate Tool(6m)

2. **Study of and experimentation** with the internal interactions of a complex system, or of a subsystem within a complex system.
3. Informational, organizational and environmental changes can be simulated and **the model's behavior can be observer.**
4. The knowledge gained in designing a simulation model can be of great **value toward suggesting improvement in the system under investigation.**
5. By changing simulation inputs and observing the resulting outputs, valuable insight may be obtained into which variables are most important and how variables interact.
6. Simulation can be used as a **pedagogical (teaching) device** to reinforce analytic solution methodologies.
7. Can be used to experiment **with new designs or policies prior to implementation,** so as to prepare for what may happen.
8. Can be used to **verify analytic solutions.**
9. By simulating different capabilities for a machine, requirements can be determined.
10. Simulation models **designed for training, allow learning without the cost and disruption of on-the-job instructions.**
11. **Animation** shows a system in simulated operation so that **the plan can be visualized**.
12. **The modern system (factory, water fabrication plant, service organization, etc) is so complex** that the interactions can be treated only through simulation

When Simulation is Not Appropriate

1. Simulation should **not be used when the problem can be solved using common sense.**
2. Not, if the problem can be **solved analytically.**
3. Not, if it is easier to perform **direct experiments.**
4. Not, if the **costs exceeds** savings.
5. Not, if the **resources or time are not available.**
6. No data is available, not even estimate simulation is not advised.
7. If there is not enough time or the people are not available, simulation is not appropriate.
8. If managers have unreasonable expectation say, too much soon – or the power of simulation is over estimated, simulation may not be appropriate.
9. **If system behavior is too complex or cannot be defined,** simulation is not appropriate.

1.2 Advantages of Simulation

1. **New policies, operating procedures, decision rules, information flow, etc can be explored** without disrupting the ongoing operations of the real system.
2. **New hardware designs, physical layouts, transportation systems** can be tested without committing resources for their acquisition.
3. **Hypotheses** about how or why certain phenomena occur can be **tested for feasibility.**

4. **Time can be compressed or expanded allowing for a speedup or slowdown** of the phenomena under investigation.
5. Insight can be obtained about the **interaction of variables.**
6. Insight can be obtained about **the importance of variables to the performance of the system.**
7. **Bottleneck analysis** can be performed indication where work-in process, information materials and so on are being excessively delayed.
8. A **simulation study can help in understanding how the system operates** rather than how individuals think the system operates.
9. "what-if" questions can be answered. **Useful in the design of new systems.**

Disadvantages of simulation

1. Model building **requires special training**. It is an art **that is learned over time and through experience.**
2. If two models are **constructed by two competent individuals,** they may have similarities, but it is highly unlikely that they will be the same.
3. Simulation results may be **difficult to interpret.** Since most simulation outputs are essentially random variables (they are usually based on random inputs), it may be hard to determine whether an observation is a result of system interrelationships or randomness.
4. Simulation modeling and analysis can be **time consuming and expensive**. Skimping on resources for modeling and analysis may result in a simulation model or analysis that is not sufficient for the task.
5. Simulation is used in some cases when an analytical solution is possible, or even preferable. This might be particularly true in the simulation of some waiting lines where closed-form queueing models are available.

1.3 Applications of Simulation

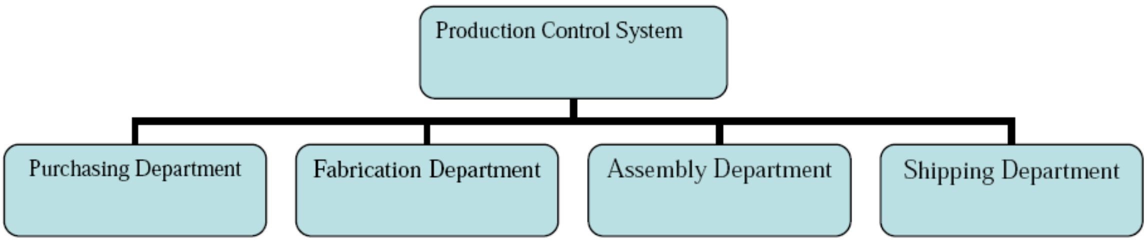
2. **Manufacturing Applications**
   - Analysis of electronics assembly operations
   - Design and evaluation of a selective assembly station for high-precision scroll compressor shells
   - Comparison of dispatching rules for semiconductor manufacturing using large-facility models
   - Evaluation of cluster tool throughput for thin-film head production
   - Determining optimal lot size for a semiconductor back-end factory
   - Optimization of cycle time and utilization in semiconductor test manufacturing
   - Analysis of storage and retrieval strategies in a warehouse
   - Investigation of dynamics in a service-oriented supply chain
   - Model for an Army chemical munitions disposal facility
3. **Semiconductor Manufacturing**
   - Comparison of dispatching rules using large-facility models
   - The corrupting influence of variability
   - A new lot-release rule for wafer fabs
   - Assessment of potential gains in productivity due to proactive reticle management
   - Comparison of a 200-mm and 300-mm X-ray lithography cell
   - Capacity planning with time constraints between operations
   - 300-mm logistic system risk reduction
4. **Construction Engineering**
   - Construction of a dam embankment
   - Trenchless renewal of underground urban infrastructures
   - Activity scheduling in a dynamic, multiproject setting
   - Investigation of the structural steel erection process
   - Special-purpose template for utility tunnel construction
5. **Military Application**
   - Modeling leadership effects and recruit type in an Army recruiting station
   - Design and test of an intelligent controller for autonomous underwater vehicles
   - Modeling military requirements for nonwarfighting operations
   - Multitrajectory performance for varying scenario sizes

- Using adaptive agent in U.S Air Force pilot retention

6. **Logistics, Transportation, and Distribution Applications**
   - Evaluating the potential benefits of a rail-traffic planning algorithm
   - Evaluating strategies to improve railroad performance
   - Parametric modeling in rail-capacity planning
   - Analysis of passenger flows in an airport terminal
   - Proactive flight-schedule evaluation
   - Logistics issues in autonomous food production systems for extended-duration space exploration
   - Sizing industrial rail-car fleets
   - Product distribution in the newspaper industry
   - Design of a toll plaza
   - Choosing between rental-car locations
   - Quick-response replenishment
7. **Business Process Simulation**
   - Impact of connection bank redesign on airport gate assignment
   - Product development program planning
   - Reconciliation of business and systems modeling
   - Personnel forecasting and strategic workforce planning
8. **Human Systems**
   - Modeling human performance in complex systems
   - Studying the human element in air traffic control
9. **Healthcare**
   - Modeling front office and patient care in ambulatory health care practices
   - Evaluating hospital operations b/n the emergency department and a medical
   - Estimating maximum capacity in an emergency room and reducing length of stay in that room.

1.4 Systems

A system is defined as an aggregation or assemblage of objects joined in some regular interaction or interdependence toward the accomplishment of some purpose.
Example: Production System
OR A system is assemblage of objects joined in regular fashion to accomplish a task.



System Environment

The external components which interact with the system and produce necessary changes are said to constitute the system environment.
Ex: In a factory system, the factors controlling arrival of orders may be considered to be outside the factory but yet a part of the system environment. When, we consider the demand and supply of goods, there is certainly a relationship between the factory output and arrival of orders.

Endogenous System:	The term endogenous is **used to describe activities and events occurring within a system.**	Ex: Drawing cash in a bank.
Exogenous	The term exogenous is **used to describe activities and events in the environment that affect the system.**	Ex: Arrival of customers.

System:		
Closed System:	A system for which there is **no exogenous activity and event** is said to be a closed.	Ex: Water in an insulated flask.
Open system:	A system for which there is **exogenous activity and event** is said to be an open.	Ex: Bank system.

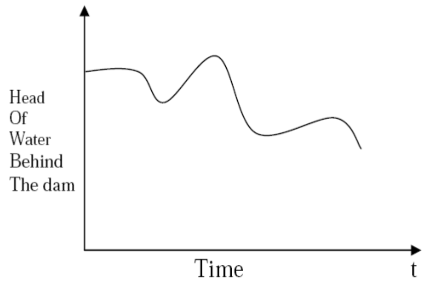
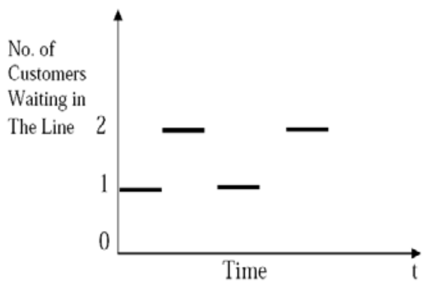
1.5 Components of a System

1) **Entity**: An entity is an object of interest in a system.
Ex: In the factory system, departments, orders, parts and products are the entities.
2) **Attribute**: An attribute denotes the property of an entity.
Ex: Quantities for each order, type of part, or number of machines in a department are attributes of factory system.
3) **Activity**: Any process causing changes in a system is called as an activity.
Ex: Manufacturing process of the department.
4) **State of the System**: The state of a system is defined as the collection of variables necessary to describe a system at any time, relative to the objective of study.
5) **Event**: An event is defined as an instantaneous occurrence that may change the state of the system.

Examples of system and components

System	Entities	Attributes	Activities	Events	State variables
Banking	Customers	Checking-account balance	Making deposits	Arrival; departure	No. of busy tellers; no. of customers waiting.
Rapid rail	Riders	Origination; destination	Traveling	Arrival at station; arrival at destination	No. of riders waiting at each station; No. of riders in transit
Production	Machines	Speed; capacity; breakdown rate length	Welding; stamping	Breakdown	Status of machines (busy, idle or down)
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; backlogged demands

1.6 Discrete and Continuous Systems

Continuous Systems	**Systems in which the changes are predominantly smooth** are called continuous system.	Ex: Head of a water behind a dam.	
Discrete Systems	**Systems in which the changes are predominantly discontinuous** are called discrete systems.	Ex: Bank – the number of customer's changes only when a customer arrives or when the service provided a customer is completed.	

1.7 Model of a system

- A **model** is defined as a representation of a system for the purpose of studying the system.
- It is necessary to consider only those aspects of the system that affect the problem under investigation.
- These aspects are represented in a model, and by definition it is a simplification of the system.

1.8 Types of Models

Sl. No	Model	Description
1	**Mathematical Model**	Uses symbolic notation and the mathematical equations to represent a system.
2	**Static Model**	Represents a system at a particular point of time and also known as Monte-Carlo simulation.
3	**Dynamic Model**	Represents systems as they change over time. Ex: Simulation of a bank
4	**Deterministic Model**	Contains no random variables. They have a known set of inputs which will result in a unique set of outputs. Ex: Arrival of patients to the Dentist at the scheduled appointment time.
5	**Stochastic Model**	Has one or more random variable as inputs. Random inputs leads to random outputs. Ex: Simulation of a bank involves random inter arrival and service times.
6	**Discrete and Continuous Model**	Used in an analogous manner. Simulation models may be mixed both with discrete and continuous. The choice is based on the characteristics of the system and the objective of the study.

1.9 Discrete-Event System Simulation

- Modeling of systems in which the state variable changes only at a discrete set of points in time. **The simulation models are analyzed by numerical rather than by analytical methods.**
- **Analytical methods** employ the deductive reasoning of mathematics to solve the model. E.g.: Differential calculus can be used to determine the minimum cost policy for some inventory models.
- **Numerical methods** use computational procedures and are 'runs', which is generated based on the model assumptions and observations are collected to be analyzed and to estimate the true system performance measures.
- Real-world simulation is so vast, whose runs are conducted with the help of computer. Much insight can be obtained by simulation manually which is applicable for small systems.

1.10 Steps in a Simulation study

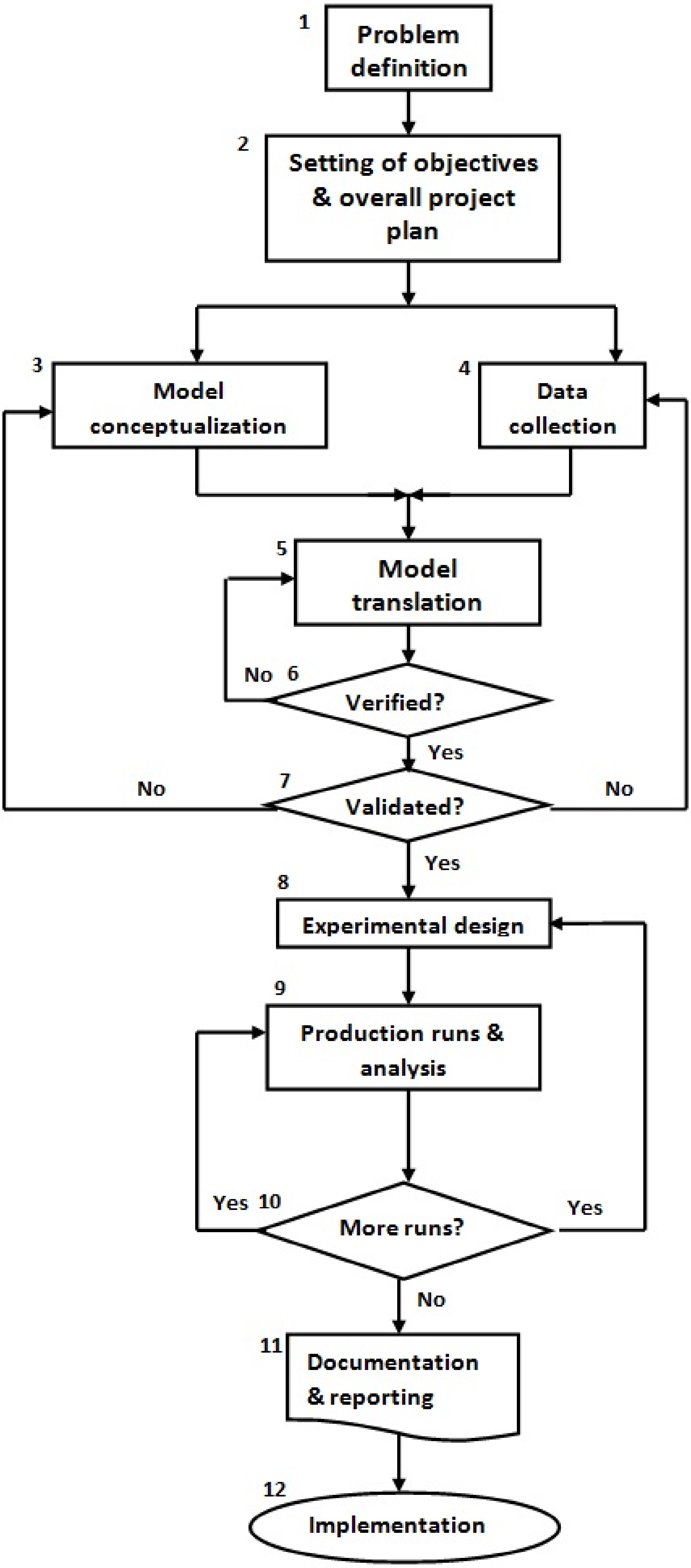
1. **Problem formulation:** Every study begins with a statement of the problem, provided by **policy makers. Analyst** ensures it's clearly understood. If it is developed by analyst and policy makers should understand and agree with it.
2. **Setting of objectives and overall project plan:** The objectives indicate the questions to be answered by simulation. At this point a determination should be made concerning whether simulation is the appropriate methodology. Assuming that it is appropriate, the overall project plan should include
   I. A statement of the alternative systems
   II. A method for evaluating the effectiveness of these alternatives
   III. Plans for the study in terms of the number of people involved
   IV. Cost of the study
   V. The number of days required to accomplish each phase of the work with the anticipated results.
3. **Model conceptualization:** The construction of a model of a system is probably as much art as science. The art of modeling is enhanced by ability.
   a. To abstract the essential features of a problem.
   b. To select and modify basic assumptions that characterizes the system.
   c. To enrich and elaborate the model until a useful approximation results.

   Thus, it is best to start with a simple model and build toward greater complexity. Model conceptualizations enhance the quality of the resulting model and increase the confidence of the model user in the application of the model.
4. **Data collection:** As the complexity of the model changes, the required data elements may also change.

5. **Model translation**: Real-world systems result in models that require a great deal of information storage and computation. It can be programmed by using simulation languages or special purpose simulation software. Simulation languages are powerful and flexible. Simulation software models development time can be reduced. GPSS/H^{TM} or special-purpose simulation software.
6. **Verified**: It pertains to the computer program and checking the performance. If the input parameters and logical structure and correctly represented, verification is completed.
7. **Validated:** It is the determination that a model is an accurate representation of the real system. Achieved through calibration of the model, an iterative process of comparing the model to actual system behavior and the discrepancies between the two.
8. **Experimental Design:** The alternatives that are to be simulated must be determined. For each system design, decisions need to be made concerning
    a. Length of the initialization period
    b. Length of simulation runs
    c. Number of replication to be made of each run
9. **Production runs and analysis**: They are used to estimate measures of performance for the system designs that are being simulated.
10. **More runs:** Based on the analysis of runs that have been completed. The analyst determines if additional runs are needed and what design those additional experiments should follow.
11. **Documentation and reporting:** Two types of documentation. Program documentation and Process documentation
    a. **Program documentation:** Can be used again by the same or different analysts to understand how the program operates
    b. **Process documentation:** This enable to review the final formulation and alternatives, results of the experiments and the recommended solution to the problem. The final report provides a vehicle of certification.
12. **Implementation:** Success depends on the previous steps. If the model user has been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.

The simulation model building process can be broken into four phases

I Phase: Consists of steps 1 and 2 • It is period of discovery/orientation • The analyst may have to restart the process if it is not fine-tuned • Recalibrations and clarifications may occur in this phase or another phase.	**II Phase:** Consists of steps 3,4,5,6 and 7 • a model building and data collection • A continuing interplay is required among the steps • Exclusion of model user results in implications during implementation
III Phase: Consists of steps 8,9 and 10 • running the model • Conceives a thorough plan for experimenting • Discrete-event stochastic is a statistical experiment • The output variables are estimates that contain random error and therefore proper statistical analysis is required.	**IV Phase:** Consists of steps 11 and 12 • an implementation • Successful implementation depends on the involvement of user and every steps successful completion.



Steps in a simulation study

1.11 Simulation Examples

Three steps of the simulations

- Determine the characteristics of each of the inputs to the simulation. Quite often, these may be **modeled as probability distributions, either continuous or discrete.**
- **Construct a simulation table**. Each simulation table is different, for each is developed for the problem at hand.
- For each repetition ***i***, generate a value for each of the p inputs, and evaluate the function, calculating a value of the response y_i. The input values may be computed by sampling values from the distributions determined in step 1. A response typically depends on the inputs and one or more previous responses.
- The simulation table provides a systematic method for tracking system state over time.

	Inputs						Response
Repetitions	X_{i1}	X_{i2}	...	X_{ij}	...	X_{ip}	y_i
1							
2							
.							
.							
.							
n							

1.12 Simulation of queueing systems

A **Queueing system** is described by **its calling population, the nature of its arrivals, the service mechanism, the system capacity, and queueing discipline.** Simulation is often used in the analysis of queueing models. In a simple typical queueing model, shown in

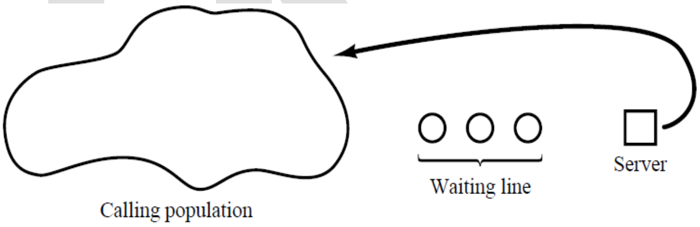


Figure 2.1 Queueing system.

- In the single-channel queue, the calling population is **infinite**; that is, if a **unit leaves the calling population and joins the waiting line or enters service, there is no change in the arrival rate of other units that may need service.**
- **Arrivals for service occur one at a time in a random fashion**; once they **join the waiting line,** they are eventually served.
- The **system capacity has no limit**, meaning that any number of units can wait in line. Finally, **units are served in the order of their arrival (often called FIFO: first in, first out) by a single server or channel**.
- **Arrivals and services** are defined by the **distributions of the time between arrivals** and the **distribution of service times,** respectively.
- For any **simple single or multi-channel queue**, the overall effective arrival rate must be less than the total service rate, or the waiting line will grow without bound. When queues grow without bound, they are termed **"explosive" or unstable.**
- The **state of the system:** the number of units in the system and the status of the server, busy or idle.
- **An event**: a set of circumstances that cause an instantaneous change in the state of the system.
  In a single-channel queueing system there are only two possible events that can affect the state of the system.
  - **the arrival event :** the entry of a unit into the system
  - **the departure event :** the completion of service on a unit

  The queueing system includes the server, the unit being serviced, and units in the queue.
- The **simulation clock** is used to track simulated time.

- If a unit has just completed service, the simulation proceeds in the manner shown in **the flow diagram of figure 2.2.**
  Note that the server has only two possible states: it is either **busy or idle.**

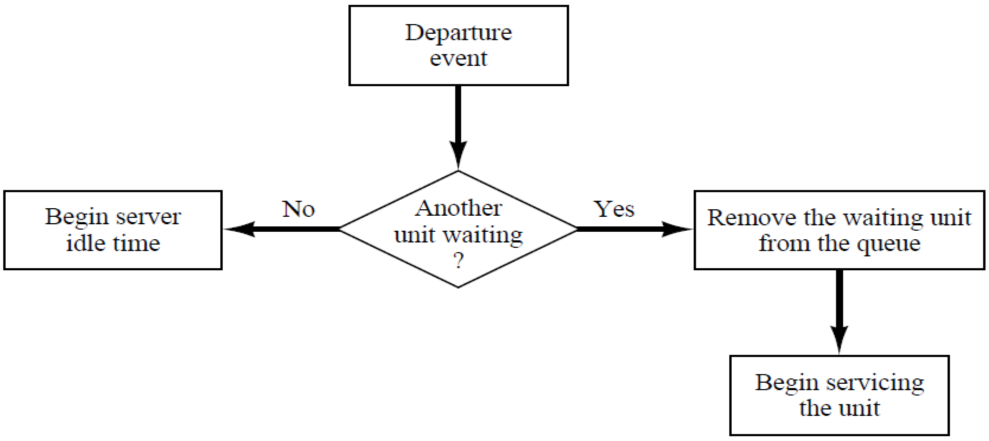


Figure 2.2 Service-just-completed flow diagram.

- The arrival event occurs when a unit enters the system. **The flow diagram for the arrival event is shown in figure 2.3**

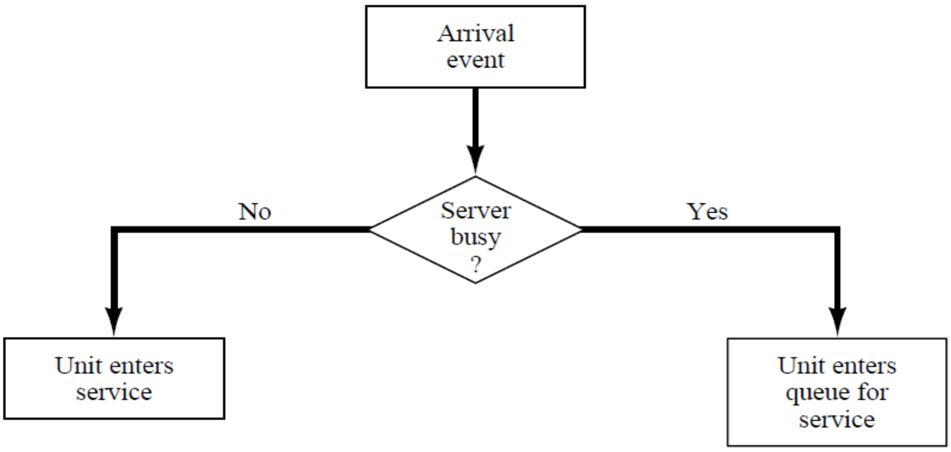


Figure 2.3 Unit-entering-system flow diagram.

- The unit may find the **server either idle or busy;** therefore, either the unit begins service immediately, or it enters the queue for the server. The unit follows the course of action shown in fig 2.4.
- If the server is busy, the unit enters the queue. If the server is idle and the queue is empty, the unit begins service. It is not possible for the server to be idle and the queue to be nonempty.

		Queue status	
		Not empty	Empty
Server status	Busy	Enter queue	Enter queue
	Idle	Impossible	Enter service

Figure 2.4 Potential unit actions upon arrival.

- After the completion of a service the service may become idle or remain busy with the next unit. The relationship of these two outcomes to the status of the queue is shown in fig 2.5.
- If the queue is not empty, another unit will enter the server and it will be busy.
- If the queue is empty, the server will be idle after a service is completed. These two possibilities are shown as the shaded portions of fig 2.5. It is impossible for the server to become busy if the queue is empty when a service is completed. Similarly, it is impossible for the server to be idle after a service is completed when the queue is not empty.

		Queue status	
		Not empty	Empty
Server outcomes	Busy		Impossible
	Idle	Impossible	

Figure 2.5 Server outcomes after service completion.

- Simulations of queueing systems generally require the maintenance of an event list for determining what happens next.
- Simulation clock times for arrivals and departures are computed in a simulation table customized for each problem.
- In simulation, events usually occur at random times, the randomness imitating uncertainty in real life.
- Random numbers are distributed uniformly and independently on the interval (0, 1).
- Random digits are uniformly distributed on the set {0, 1, 2… 9}.
- The proper number of digits is dictated by the accuracy of the data being used for input purposes.
- Pseudo-random numbers: the numbers are generated using a procedure.
- **Table 2.2. Interarrival and Clock Times**
    - Assume that the times between arrivals were generated by rolling a die five times and recording the up face.

Table 2.2 Interarrival and Clock Times

Customer	*Interarrival Time*	*Arrival Time on Clock*
1	–	0
2	2	2
3	4	6
4	1	7
5	2	9
6	6	15

- **Table 2.3. Service Times**
    - Assuming that all four values are equally likely to occur, these values could have been generated by placing the numbers one through four on chips and drawing the chips from a hat with replacement, being sure to record the numbers selected.
    - The only possible service times are one, two, three, and four time units.

Table 2.3 Service Times

Customer	*Service Time*
1	2
2	1
3	3
4	2
5	1
6	4

- The interarrival times and service times must be meshed to simulate the single-channel queueing system.
- Table 2.4 was designed specifically for a single-channel queue which serves customers on a first-in, first-out (FIFO) basis.

Table 2.4 Simulation Table Emphasizing Clock Times

A Customer Number	B Arrival Time (Clock)	C Time Service Begins (Clock)	D Service Time (Duration)	E Time Service Ends (Clock)
1	0	0	2	2
2	2	2	1	3
3	6	6	3	9
4	7	9	2	11
5	9	11	1	12
6	15	15	4	19

- Table 2.4 keeps track of the clock time at which each event occurs.
- The occurrence of the two types of events (arrival and departure event) in chronological order is shown in Table 2.5 and Figure 2.6.
- Figure 2.6 is a visual image of the event listing of Table 2.5.

Table 2.5 Chronological Ordering of Events

Event Type	Customer Number	Clock Time
Arrival	1	0
Departure	1	2
Arrival	2	2
Departure	2	3
Arrival	3	6
Arrival	4	7
Departure	3	9
Arrival	5	9
Departure	4	11
Departure	5	12
Arrival	6	15
Departure	6	19

- Figure 2.6 depicts the number of customers in the system at the various clock times.

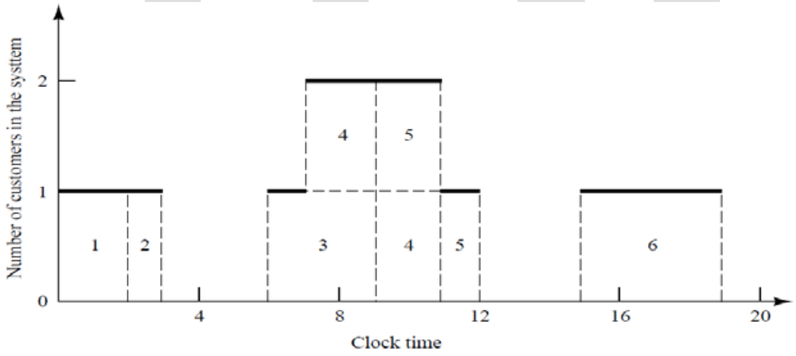
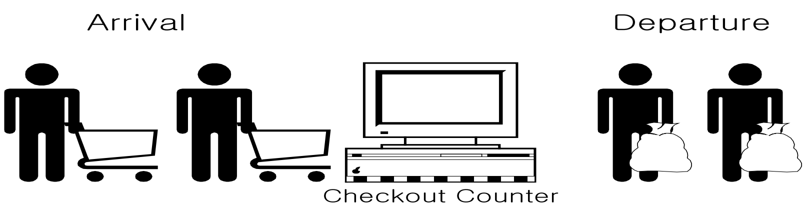


Figure 2.6 Number of customers in the system.

Example 2.1 Single-Channel Queue



- Assumptions
  - A grocery store has only one checkout counter.
  - Customers arrive at this checkout counter at random from 1 to 8 minutes apart. Each possible value of interarrival time has the same probability of occurrence, as shown in Table 2.6.
  - The service times vary from 1 to 6 minutes with the probabilities shown in Table 2.7.

- The problem is to analyze the system by simulating the arrival and service of 20 customers.

Table 2.6 Distribution of Time Between Arrivals

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.125	0.125	001–125
2	0.125	0.250	126–250
3	0.125	0.375	251–375
4	0.125	0.500	376–500
5	0.125	0.625	501–625
6	0.125	0.750	626–750
7	0.125	0.875	751–875
8	0.125	1.000	876–000

Table 2.7 Service-Time Distribution

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.10	0.10	01–10
2	0.20	0.30	11–30
3	0.30	0.60	31–60
4	0.25	0.85	61–85
5	0.10	0.95	86–95
6	0.05	1.00	96–00

- A simulation of a grocery store that starts with an empty system is not realistic unless the intention is to model the system from startup or to model until steady-state operation is reached.
- A set of uniformly distributed random numbers is needed to generate the arrivals at the checkout counter. Random numbers have the following properties:
    1. The set of random numbers is uniformly distributed between 0 and 1.
    2. Successive random numbers are independent.
- Random digits are converted to random numbers by placing a decimal point appropriately.
- The rightmost two columns of Tables 2.6 and 2.7 are used to generate random arrivals and random service times.
- The first random digits are 913. To obtain the corresponding time between arrivals, enter the fourth column of Table 2.6 and read 8 minutes from the first column of the table.

Table 2.8 Time-Between-Arrivals Determination

Customer	*Random Digits*	*Time between Arrivals (Minutes)*	*Customer*	*Random Digits*	*Time between Arrivals (Minutes)*
1	–	–	11	109	1
2	913	8	12	093	1
3	727	6	13	607	5
4	015	1	14	738	6
5	948	8	15	359	3
6	309	3	16	888	8
7	922	8	17	106	1
8	753	7	18	212	2
9	235	2	19	493	4
10	302	3	20	535	5

- The first customer's service time is 4 minutes because the random digits 84 fall in the bracket 61-85

Table 2.9 Service Times Generated

Customer	*Random Digits*	*Service Time (Minutes)*	*Customer*	*Random Digits*	*Service Time (Minutes)*
1	84	4	11	32	3
2	10	1	12	94	5
3	74	4	13	79	4
4	53	3	14	05	1
5	17	2	15	79	5
6	79	4	16	84	4
7	91	5	17	52	3
8	67	4	18	55	3
9	89	5	19	30	2
10	38	3	20	50	3

- The essence of a manual simulation is the simulation table.

- The simulation table for the single-channel queue, shown in Table 2.10, is an extension of the type of table already seen in Table 2.4.
- Statistical measures of performance can be obtained from the simulation table such as Table 2.10.
- Statistical measures of performance in this example.
    - o Each customer's time in the system
    - o The server's idle time
- In order to compute summary statistics, totals are formed as shown for service times, time customers spend in the system, idle time of the server, and time the customers wait in the queue.

Table 2.10 Simulation Table for Queueing Problem

A Customer	B *Time Since Last Arrival (Minutes)*	C *Arrival Time*	D *Service Time (Minutes)*	E *Time Service Begins*	F *Time Customer Waits in Queue (Minutes)*	G *Time Service Ends*	H *Time Customer Spends in System (Minutes)*	I *Idle Time of Server (Minutes)*
1	–	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
11	1	47	3	53	6	56	9	0
12	1	48	5	56	8	61	13	0
13	5	53	4	61	8	65	12	0
14	6	59	1	65	6	66	7	0
15	3	62	5	66	4	71	9	0
16	8	70	4	71	1	75	5	0
17	1	71	3	75	4	78	7	0
18	2	73	3	78	5	81	8	0
19	4	77	2	81	4	83	6	0
20	5	82	3	83	1	86	4	0
			68		56		124	18

- The average waiting time for a customer : 2.8 minutes

$$average\ waitng\ time = \frac{total\ time\ customers\ wait\ in\ queue}{total\ numbers\ of\ customers} = \frac{56}{20} = 2.8\,(\mathrm{min})$$

- The probability that a customer has to wait in the queue : 0.65

$$probability\ (wait) = \frac{number\ of\ customers\ who\ wait}{total\ numbers\ of\ customers} = \frac{13}{20} = 0.65$$

- The fraction of idle time of the server : 0.21

$$probability\ of\ idle\ server = \frac{total\ idle\ time\ of\ server}{total\ run\ time\ of\ simulation} = \frac{18}{86} = 0.21$$

- The average service time : 3.4 minutes

$$average\ service\ time = \frac{total\ service\ time}{total\ numbers\ of\ customers} = \frac{68}{20} = 3.4\,(\mathrm{min})$$

This result can be compared with the expected service time by finding the mean of the service-time distribution using the equation in table 2.7.

$$E(S) = \sum_{s=0}^{\infty} sp(s)$$

$$E(S) = 1(0.10) + 2(0.20) + 3(0.30) + 4(0.25) + 5(1.10) + 6(0.05) = 3.2 \text{ (min)}$$

The expected service time is slightly lower than the average service time in the simulation. The longer the simulation, the closer the average will be to $E(S)$

- The average time between arrivals : 4.3 minutes

$$\textit{average time between arrivals} = \frac{\textit{sum of all times between arrivals}}{\textit{numbers of arrivals} - 1} = \frac{82}{19} = 4.3 \text{ (min)}$$

- This result can be compared to the expected time between arrivals by finding the mean of the discrete uniform distribution whose endpoints are a=1 and b=8.

$$E(A) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5 \text{ (min)}$$

The longer the simulation, the closer the average will be to $E(A)$

- The average waiting time of those who wait : 4.3 minutes

$$\textit{average waiting time of those who wait} = \frac{\textit{total time customers wait in queue}}{\textit{total numbers of customers who wiat}} = \frac{56}{13} = 4.3 \text{ (min)}$$

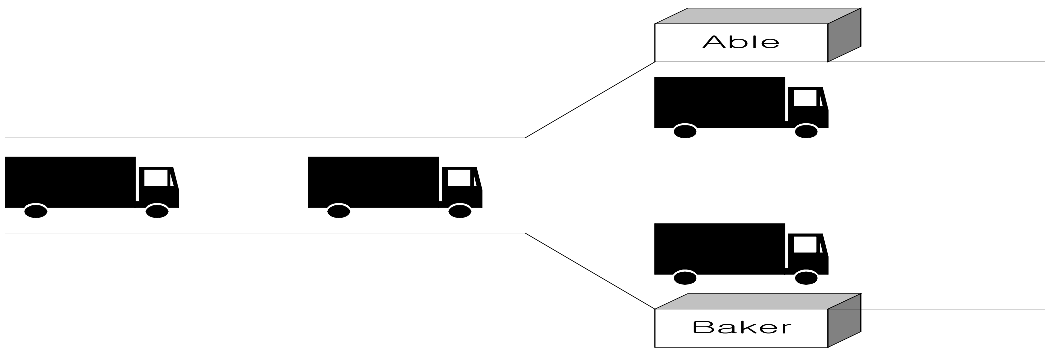
- The average time a customer spends in the system : 6.2 minutes

$$\textit{average time customer spends in the system} = \frac{\textit{total time customers spend in system}}{\textit{total numbers of customers}} = \frac{124}{20} = 6.2 \text{ (min)}$$

Average time customer spends in the system	=	Average time customer spends waiting in the	+	Average time customer spends in service

$\therefore$ Average time customer spends in the system = 2.8 + 3.4 = 6.2 (min)

Example 2.2 the Able Baker Carhop Problem



- A drive-in restaurant where carhops take orders and bring food to the car.
- **Assumptions**
  - Cars arrive in the manner shown in Table 2.11.
  - Two carhops Able and Baker - Able is better able to do the job and works a bit faster than Baker.
  - The distribution of their service times is shown in Tables 2.12 and 2.13.
  - A simplifying rule is that Able gets the customer if both carhops are idle.
  - If both are busy, the customer begins service with the first server to become free.
  - To estimate the system measures of performance, a simulation of 1 hour of operation is made.
  - The problem is to find how well the current arrangement is working.

Table 2.11 Interarrival Distribution of Cars

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.25	0.25	01–25
2	0.40	0.65	26–65
3	0.20	0.85	66–85
4	0.15	1.00	86–00

Table 2.12 Service Distribution of Able

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Table 2.13 Service Distribution of Baker

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

Table 2.14 Simulation Table for Carhop Example

A	B	C	D	E	F	G	H	I	J	K	L
						Able			*Baker*		
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1
7	74	3	17	13	18	2	20				1
8	80	3	20	61	20	4	24				0
9	68	3	23	50				23	4	27	0
10	22	1	24	49	24	3	27				0
11	48	2	26	39	27	3	30				1
12	34	2	28	53				28	4	32	0
13	45	2	30	88	30	5	35				0
14	24	1	31	01				32	3	35	1
15	34	2	33	81	35	4	39				2
16	63	2	35	53				35	4	39	0
17	38	2	37	81	39	4	43				2
18	80	3	40	64				40	5	45	0
19	42	2	42	01	43	2	45				1
20	56	2	44	67	45	4	49				1
21	89	4	48	01				48	3	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			43		11

- **The analysis of Table 2.14 results in the following:**
  - Over the 62-minute period Able was busy 90% of the time.
  - Baker was busy only 69% of the time. The seniority rule keeps Baker less busy (and gives Able more tips).
  - Nine of the 26 arrivals (about 35%) had to wait. The average waiting time for all customers was only about 0.42 minute (25 seconds), which is very small.
  - Those nine who did have to wait only waited an average of 1.22 minutes, which is quite low.
  - In summary, this system seems well balanced. One server cannot handle all the diners, and three servers would probably be too many. Adding an additional server would surely reduce the waiting time to nearly zero. However, the cost of waiting would have to be quite high to justify an additional server.

1.13 Simulation of Inventory Systems

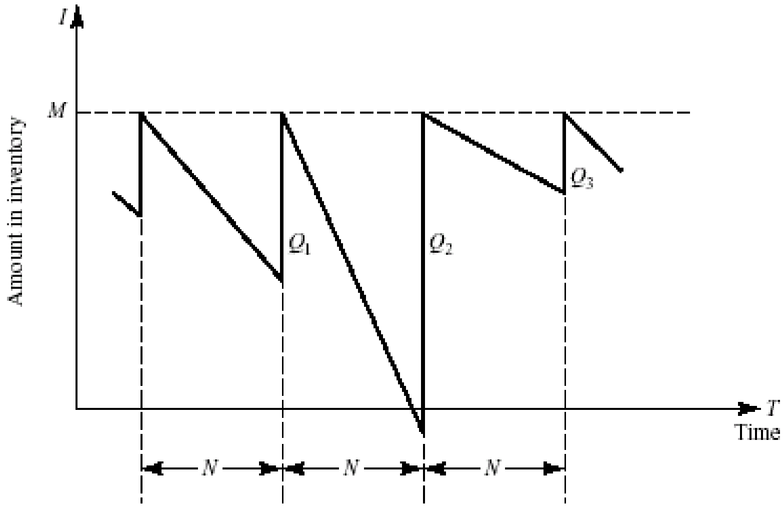


Figure 2.7 Probabilistic order-level inventory system.

- This inventory system has a periodic review of length N, at which time the inventory level is checked.
- An order is made to bring the inventory up to the level M.
- In this inventory system the **<u>lead time</u>** (i.e., the length of time between the placement and receipt of an order) is zero.
- **<u>Demand</u>** is shown as being uniform over the time period
- Notice that in the second cycle, the amount in inventory drops below zero, indicating a **<u>shortage</u>**.
- **<u>Two way to avoid shortages</u>**
  - **Carrying stock in inventory:** Cost - the interest paid on the funds borrowed to buy the items, renting of storage space, hiring guards, and so on.
  - **Making more frequent reviews, and consequently, more frequent purchases or replenishments** : the ordering cost
- The total cost of an inventory system is the **<u>measure of performance.</u>**
  - The decision maker can control the maximum inventory level, M, and the length of the cycle, N.
  - In an (M, N) inventory system, the events that may occur are: the demand for items in the inventory, the review of the inventory position, and the receipt of an order at the end of each review period.

$$Pofit = \left[\begin{pmatrix} revenue \\ from\ sales \end{pmatrix} - \begin{pmatrix} \text{cost } of \\ newspapers \end{pmatrix} - \begin{pmatrix} lost\ \ profit\ \ from \\ excess\ demand \end{pmatrix} + \begin{pmatrix} salvage\ \ from\ sale \\ of\ \ scrap\ \ papers \end{pmatrix} \right]$$

  - The problem is to determine the optimal number of papers the newspaper seller should purchase.
  - This will be accomplished by simulating demands for 20 days and recording profits from sales each day.
  - The profits are given by the following relationship:
  - The distribution of papers demanded on each of these days is given in Table 2.15.
  - Tables 2.16 and 2.17 provide the random-digit assignments for the types of newsdays and the demands for those newsdays.

Table 2.15 Distribution of Newspapers Demanded

	Demand Probability Distribution		
Demand	*Good*	*Fair*	*Poor*
40	0.03	0.10	0.44
50	0.05	0.18	0.22
60	0.15	0.40	0.16
70	0.20	0.20	0.12
80	0.35	0.08	0.06
90	0.15	0.04	0.00
100	0.07	0.00	0.00

Table 2.16 Random-Digit Assignment for Type of Newsday

Type of Newsday	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
Good	0.35	0.35	01–35
Fair	0.45	0.80	36–80
Poor	0.20	1.00	81–00

Table 2.17 Random-Digit Assignments for Newspapers Demanded

	Cumulative Distribution			Random-Digit Assignment		
Demand	*Good*	*Fair*	*Poor*	*Good*	*Fair*	*Poor*
40	0.03	0.10	0.44	01–03	01–10	01–44
50	0.08	0.28	0.66	04–08	11–28	45–66
60	0.23	0.68	0.82	09–23	29–68	67–82
70	0.43	0.88	0.94	24–43	69–88	83–94
80	0.78	0.96	1.00	44–78	89–96	95–00
90	0.93	1.00	1.00	79–93	97–00	
100	1.00	1.00	1.00	94–00		

Table 2.18 Simulation Table for Purchase of 70 Newspapers

Day	*Random Digits for Type of Newsday*	*Type of Newsday*	*Random Digits for Demand*	*Demand*	*Revenue from Sales*	*Lost Profit from Excess Demand*	*Salvage from Sale of Scrap*	*Daily Profit*
1	94	Poor	80	60	$30.00	–	$0.50	$7.40
2	77	Fair	20	50	25.00	–	1.00	2.90
3	49	Fair	15	50	25.00	–	1.00	2.90
4	45	Fair	88	70	35.00	–	–	11.90
5	43	Fair	98	90	35.00	$3.40	–	8.50
6	32	Good	65	80	35.00	1.70	–	10.20
7	49	Fair	86	70	35.00	–	–	11.90
8	00	Poor	73	60	30.00	–	0.50	7.40
9	16	Good	24	70	35.00	–	–	11.90
10	24	Good	60	80	35.00	1.70	–	10.20
11	31	Good	60	80	35.00	1.70	–	10.20
12	14	Good	29	70	35.00	–	–	11.90
13	41	Fair	18	50	25.00	–	1.00	2.90
14	61	Fair	90	80	35.00	1.70	–	10.20
15	85	Poor	93	70	35.00	–	–	11.90
16	08	Good	73	80	35.00	1.70	–	10.20
17	15	Good	21	60	30.00	–	0.50	7.40
18	97	Poor	45	50	25.00	–	1.00	2.90
19	52	Fair	76	70	35.00	–	–	11.90
20	78	Fair	96	80	35.00	1.70	–	10.20
					$645.00	$13.60	$5.50	$174.90

- The simulation table for the decision to purchase 70 newspapers is shown in Table 2.18.
- The profit for the first day is determined as follows:
  - **Profit = $30.00 - $23.10 - 0 + $.50 = $7.40**
    - On day 1 the demand is for 60 newspapers. The revenue from the sale of 60 newspapers is $30.00.
    - Ten newspapers are left over at the end of the day.
    - The salvage value at 5 cents each is 50 cents.

- The profit for the 20-day period is the sum of the daily profits, \$174.90. It can also be computed from the totals for the 20 days of the simulation as follows:
  Total profit = \$645.00 - \$462.00 - \$13.60 + \$5.50 = \$174.90
- The policy (number of newspapers purchased) is changed to other values and the simulation repeated until the best value is found.

Example 2.4 Simulation of an (M,N) Inventory System

- This example follows the pattern of the probabilistic order-level inventory system shown in Figure 2.7.
- Suppose that the maximum inventory level, M, is 11 units and the review period, N, is 5 days. The problem is to estimate, by simulation, the average ending units in inventory and the number of days when a shortage condition occurs.
- The distribution of the number of units demanded per day is shown in Table 2.19.
- In this example, lead time is a random variable, as shown in Table 2.20.
- Assume that orders are placed at the close of business and are received for inventory at the beginning of business as determined by the lead time.
- For purposes of this example, only five cycles will be shown.
- The random-digit assignments for daily demand and lead time are shown in the rightmost columns of Tables 2.19 and 2.20.

Table 2.19 Random-Digit Assignments for Daily Demand

Demand	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
0	0.10	0.10	01–10
1	0.25	0.35	11–35
2	0.35	0.70	36–70
3	0.21	0.91	71–91
4	0.09	1.00	92–00

Table 2.20 Random-Digit Assignments for Lead Time

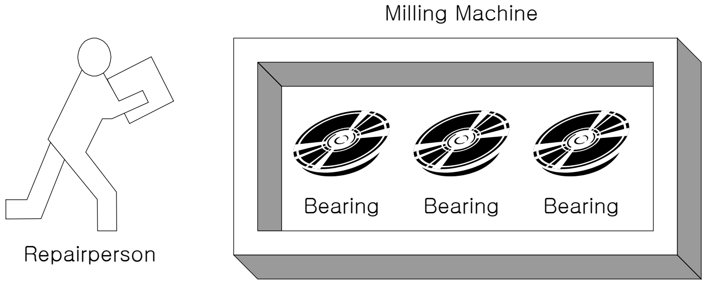
Lead Time (Days)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.6	0.6	1–6
2	0.3	0.9	7–9
3	0.1	1.0	0

Table 2.21 Simulation Tables for (M, N) Inventory System

Cycle	*Day*	*Beginning Inventory*	*Random Digits for Demand*	*Demand*	*Ending Inventory*	*Shortage Quantity*	*Order Quantity*	*Random Digits for Lead Time*	*Days until Order Arrives*
1	1	3	24	1	2	0	–	–	1
	2	2	35	1	1	0	–	–	0
	3	9	65	2	7	0	–	–	–
	4	7	81	3	4	0	–	–	–
	5	4	54	2	2	0	9	5	1
2	1	2	03	0	2	0	–	–	0
	2	11	87	3	8	0	–	–	–
	3	8	27	1	7	0	–	–	–
	4	7	73	3	4	0	–	–	–
	5	4	70	2	2	0	9	0	3
3	1	2	47	2	0	0	–	–	2
	2	0	45	2	0	2	–	–	1
	3	0	48	2	0	4	–	–	0
	4	9	17	1	4	0	–	–	–
	5	4	09	0	4	0	7	3	1
4	1	4	42	2	2	0	–	–	0
	2	9	87	3	6	0	–	–	–
	3	6	26	1	5	0	–	–	–
	4	5	36	2	3	0	–	–	–
	5	3	40	2	1	0	10	4	1
5	1	1	07	0	1	0	–	–	0
	2	11	63	2	9	0	–	–	–
	3	9	19	1	8	0	–	–	–
	4	8	88	3	5	0	–	–	–
	5	5	94	4	1	0	10	8	2
					88				

1.14 Other Examples of Simulation

Example 2.5 A Reliability Problem



- Downtime for the mill is estimated at $5 per minute.
- The direct on-site cost of the repairperson is $15 per hour.
- It takes 20 minutes to change one bearing, 30 minutes to change two bearings, and 40 minutes to change three bearings.
- The bearings cost $16 each.
- A proposal has been made to replace all three bearings whenever a bearing fails.

Table 2.22 Bearing-Life Distribution

Bearing Life (Hours)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1000	0.10	0.10	01−10
1100	0.13	0.23	11−23
1200	0.25	0.48	24−48
1300	0.13	0.61	49−61
1400	0.09	0.70	62−70
1500	0.12	0.82	71−82
1600	0.02	0.84	83−84
1700	0.06	0.90	85−90
1800	0.05	0.95	91−95
1900	0.05	1.00	96−00

- **The cumulative distribution function of the life of each bearing is identical, as shown in Table 2.22.**

Table 2.23 Delay-Time Distribution

Delay Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
5	0.6	0.6	1−6
10	0.3	0.9	7−9
15	0.1	1.0	0

- **The delay time of the repairperson's arriving at the milling machine is also a random variable, with the distribution given in Table 2.23.**

Table 2.24 Bearing Replacement Using Current Method

	Bearing 1					Bearing 2					Bearing 3				
	RD[a]	Life (Hours)	Accumulated Life (Hours)	RD	Delay (Minutes)	RD	Life (Hours)	Accumulated Life (Hours)	RD	Delay (Minutes)	RD	Life (Hours)	Accumulated Life (Hours)	RD	Delay (Minutes)
1	67	1,400	1,400	2	5	70	1,500	1,500	0	15	76	1,500	1,500	0	15
2	08	1,000	2,400	3	5	43	1,200	2,700	7	10	65	1,400	2,900	2	5
3	49	1,300	3,700	1	5	86	1,700	4,400	3	5	61	1,400	4,300	7	10
4	84	1,600	5,300	7	10	93	1,800	6,200	1	5	96	1,900	6,200	1	5
5	44	1,200	6,500	8	10	81	1,600	7,800	2	5	65	1,400	7,600	3	5
6	30	1,200	7,700	1	5	44	1,200	9,000	8	10	56	1,300	8,900	3	5
7	10	1,000	8,700	2	5	19	1,100	10,100	1	5	11	1,100	10,000	6	5
8	63	1,400	10,100	8	10	51	1,300	11,400	1	5	86	1,700	11,700	3	5
9	02	1,000	11,100	3	5	45	1,300	12,700	7	10	57	1,300	13,000	1	5
10	02	1,000	12,100	8	10	12	1,100	13,800	8	5	49	1,300	14,300	4	5
11	77	1,500	13,600	7	10	48	1,300	15,100	0	15	36	1,200	15,500	8	10
12	59	1,300	14,900	5	5	09	1,000	16,100	8	10	44	1,200	16,700	2	5
13	23	1,100	16,000	5	5	44	1,200	17,300	1	5	94	1,800	18,500	1	5
14	53	1,300	17,300	9	10	46	1,200	18,500	2	5	78	1,500	20,000	7	10
15	85	1,700	19,000	6	5	40	1,200	19,700	8	10					
16	75	1,500	20,500	4	5	52	1,300	21,000	5	5					
					110					125					95

[a]RD, random digits.

The cost of the current system is estimated as follows:

Cost of bearings = 46 bearings × $16/bearing = $736

Cost of delay time = (110 + 125 + 95) minutes × $5/minute = $1650

Cost of downtime during repair = 46 bearings × 20 minutes/bearing × $5/minute = $4600

Cost of repairpersons = 46 bearings × 20 minutes/bearing × $15/60 minutes = $230

Total cost = $736 + $1650 + $4600 + $230 = $7216

- Table 2.25 is a simulation using the proposed method. Notice that bearing life is taken from Table 2.24, so that for as many bearings as were used in the current method, the bearing life is identical for both methods

Table 2.25 Bearing Replacement using Proposed Method

	Bearing 1 Life (Hours)	Bearing 2 Life (Hours)	Bearing 3 Life (Hours)	First Failure (Hours)	Accumulated Life (Hours)	RD	Delay (Minutes)
1	1,400	1,500	1,500	1,400	1,400	3	5
2	1,000	1,200	1,400	1,000	2,400	7	10
3	1,300	1,700	1,400	1,300	3,700	5	5
4	1,600	1,800	1,900	1,600	5,300	1	5
5	1,200	1,600	1,400	1,200	6,500	4	5
6	1,200	1,200	1,300	1,200	7,700	3	5
7	1,000	1,100	1,100	1,000	8,700	7	10
8	1,400	1,300	1,700	1,300	10,000	8	10
9	1,000	1,300	1,300	1,000	11,000	8	10
10	1,000	1,100	1,300	1,000	12,000	3	5
11	1,500	1,300	1,200	1,200	13,200	2	5
12	1,300	1,000	1,200	1,000	14,200	4	5
13	1,100	1,200	1,800	1,100	15,300	1	5
14	1,300	1,200	1,500	1,200	16,500	6	5
15	1,700	1,200	63/1,400	1,200	17,700	2	5
16	1,500	1,300	21/1,100	1,100	18,800	7	10
17	85/1,700	53/1,300	23/1,100	1,100	19,900	0	15
18	05/1,000	29/1,200	51/1,300	1,000	20,900	5	5
							125

- Since the proposed method uses more bearings than the current method, the second simulation uses new random digits for generating the additional lifetimes.

- The random digits that lead to the lives of the additional bearings are shown above the slashed line beginning with the 15th replacement of bearing 3.
- The total cost of the new policy :

  Cost of bearings = 54 bearings × \$16/bearing = \$864

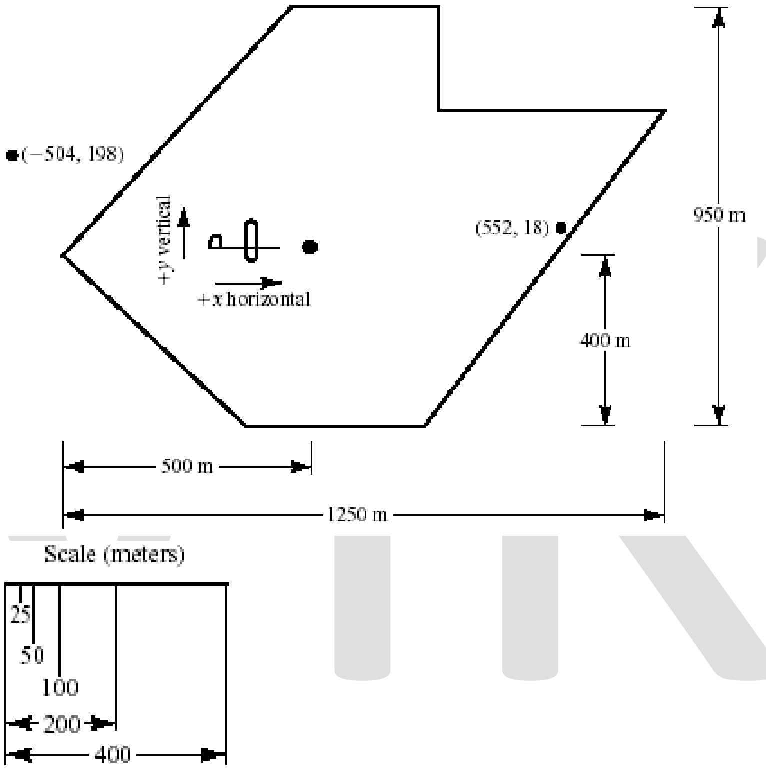
  Cost of delay time = 125 minutes × \$5/minute = \$625

  Cost of downtime during repairs = 18 sets × 40 minutes/set × \$5/minute = \$3600

  Cost of repairpersons = 18 sets × 40 minutes/set × \$15/60 minutes = \$180

  Total cost = \$864 + \$625 + \$3600 + \$180 = \$5269
- The new policy generates a savings of \$1947 over a 20,000-hour simulation. If the machine runs continuously, the simulated time is about 2 1/4 years. Thus, the savings are about \$865 per year.

Example 2.6 Random Normal Numbers



- A classic simulation problem is that of a squadron of bombers attempting to destroy an ammunition depot shaped as shown in Figure 2.8.
- If a bomb lands anywhere on the depot, a hit is scored. Otherwise, the bomb is a miss.
- The aircraft fly in the horizontal direction.
- Ten bombers are in each squadron.
- The aiming point is the dot located in the heart of the ammunition dump.
- The point of impact is assumed to be normally distributed around the aiming point with a standard deviation of 600 meters in the horizontal direction and 300 meters in the vertical direction.
- The problem is to simulate the operation and make statements about the number of bombs on target.
- The standardized normal variate, Z, with mean 0 and standard deviation 1, is distributed as

$$Z = \frac{X - \mu}{\sigma}$$

where X is a normal random variable, **μ** is the true mean of the distribution of X, and σ is the standard deviation of X.

$$X = Z\sigma + \mu$$

In this example the aiming point can be considered as (0, 0); that is, the **μ** value in the horizontal direction is 0, and similarly for the **μ** value in the vertical direction.

$$X = Z\sigma_X \qquad Y = Z\sigma_Y$$

where (X,Y) are the simulated coordinates of the bomb after it has fallen

$$\sigma_X = 600 \quad \text{and} \quad \sigma_Y = 300$$

$$X = 600Z_i \qquad Y = 300Z_i$$

The values of Z are random normal numbers.

- These can be generated from uniformly distributed random numbers.
- Alternatively, tables of random normal numbers have been generated. A small sample of random normal numbers is given in Table A.2.

The table of random normal numbers is used in the same way as the table of random numbers.

Table 2.26 shows the results of a simulated run.

Table 2.26 Simulated Bombing Run

Bomber	RNN_x	x Coordinate (600 RNN_x)	RNN_y	y Coordinate (300 RNN_y)	Result[a]
1	−0.84	−504	0.66	198	Miss
2	1.03	618	−0.13	−39	Miss
3	0.92	552	0.06	18	Hit
4	−1.82	−1,092	−1.40	−420	Miss
5	−0.16	−96	0.23	69	Hit
6	−1.78	−1,068	1.33	399	Miss
7	2.04	1,224	0.69	207	Miss
8	1.08	648	−1.10	−330	Miss
9	−1.50	−900	−0.72	−216	Miss
10	−0.42	−252	−0.60	−180	Hit

[a]Total: 3 hits, 7 misses.

- The mnemonic **RNN_x** stand for random normal number to compute the x coordinate and corresponds to **Z_i** above.
- The first random normal number used was –0.84, generating an x coordinate 600(-0.84) = -504.
- The random normal number to generate the y coordinate was 0.66, resulting in a y coordinate of 198.
- Taken together, (-504, 198) is a miss, for it is off the target.
- The resulting point and that of the third bomber are plotted on Figure 2.8.
- The 10 bombers had 3 hits and 7 misses.
- Many more runs are needed to assess the potential for destroying the dump.
- This is an example of a Monte Carlo, or static, simulation, since time is not an element of the solution.

Example 2.7 Lead-Time Demand

- Lead-time demand may occur in an inventory system.
- The lead time is the time from placement of an order until the order is received.
- In a realistic situation, lead time is a random variable.
- During the lead time, demands also occur at random. Lead-time demand is thus a random variable defined as the sum of the demands over the lead time, or

$$\sum_{i=0}^{T} D_i$$

  where i is the time period of the lead time, i = 0, 1, 2, … , Di is the demand during the ith time period; and T is the lead time.
- The distribution of lead-time demand is determined by simulating many cycles of lead time and building a histogram based on the results.
- The daily demand is given by the following probability distribution:

Daily Demand (Rolls)	3	4	5	6
Probability	0.20	0.35	0.30	0.15

The lead time is a random variable given by the following distribution:

Lead Time (Days)	1	2	3
Probability	0.36	0.42	0.22

Table 2.27 Random-Digit Assignment for Demand

Daily Demand	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
3	0.20	0.20	01−20
4	0.35	0.55	21−55
5	0.30	0.85	56−85
6	0.15	1.00	86−00

Table 2.28 Random-Digit Assignment for Lead Time

Lead Time (Days)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.36	0.36	01−36
2	0.42	0.78	37−78
3	0.22	1.00	79−00

Table 2.29 Simulation Table for Lead-Time Demand

Cycle	*Random Digits for Lead Time*	*Lead Time (Days)*	*Random Digits for Demand*	*Demand*	*Lead-Time Demand*
1	57	2	87	6	
			34	4	10
2	33	1	82	5	5
3	93	3	28	4	
			19	3	
			63	5	12
4	55	2	91	6	
			26	4	10
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

The incomplete simulation table is shown in Table 2.29.

The random digits for the first cycle were 57. This generates a lead time of 2 days.
Thus, two pairs of random digits must be generated for the daily demand.

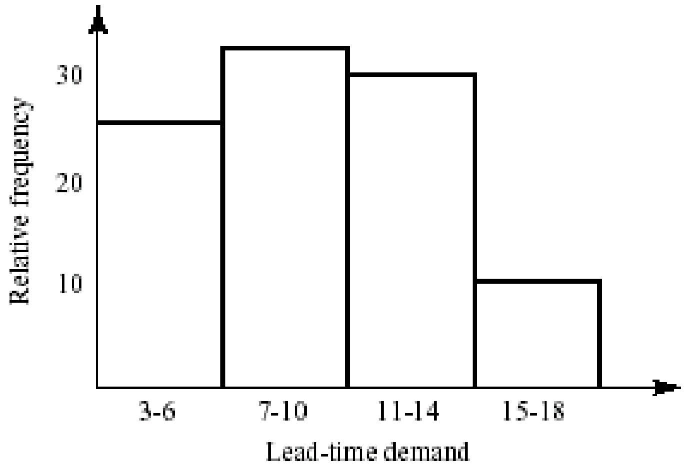


Figure 2.9 Histogram for lead-time demand.

The histogram might appear as shown in Figure 2.9.

This example illustrates how simulation can be used to study an unknown distribution by generating a random sample from the distribution.