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**06CS42**

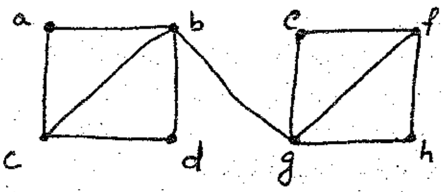
# Fourth Semester B.E. Degree Examination, May/June 2010
## Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer any FIVE full questions.***

1 a. Let G = (V, E) be the undirected graph in the Fig.1(a). How many paths are there in G from a to h? How many of these paths have a length 5? **(07 Marks)**

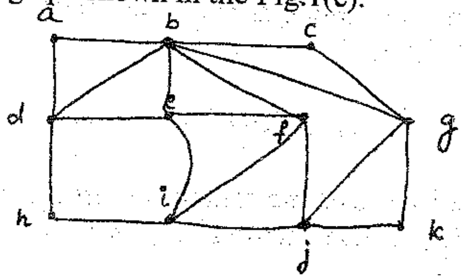
Fig.1(a).




b. Let G = (V, E) be an undirected graph, where $|V| \geq 2$. If every induced subgraph of G is connected, can we identify the graph G? **(06 Marks)**

c. Find an Euler circuit for the graph shown in the Fig.1(c). **(07 Marks)**

Fig.1(c).




2 a. Show that when any edge is removed from $K_5$, the resulting subgraph is planar. Is this true for the graph $K_{3,3}$? **(07 Marks)**

b. Nineteen students in a nursery school, play a game each day, where, they hold hands to form a circle. For how many days can they do this, with no student holding hands with the same playmate twice? **(07 Marks)**

c. Define chromatic number. What is chromatic polynomial? State the decomposition theorem for chromatic polynomials. **(06 Marks)**

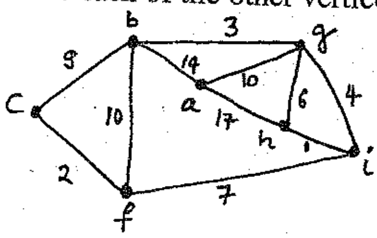
3 a. A classroom contains 25 microcomputers, that must be connected to a wall socket that has four outlets. Connections are made by using extension cords, that have four outlets each. What is the least number cords needed to get these computers set up for class use? **(07 Marks)**

b. Explain the steps in the merge sort algorithm. **(06 Marks)**

c. Using the weights 2, 3, 5, 10, 10, show that the height of the Huffman tree for a given set of weights is not unique. **(07 Marks)**

4 a. Apply Dijkstra algorithm to the weighted graph G = (V, E) shown in Fig.4(a) and determine the shortest distance from vertex a to each of the other vertices in the graph. **(07 Marks)**

Fig.4(a)




Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Use Prim's algorithm to generate an optimal tree for the graph, shown in Fig.4(b). **(06 Marks)**

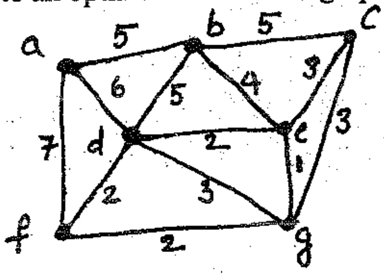



Fig.4(b).

c. Let f be a flow in a network N = (V, E). If C = (P, $\overline{P}$) is any cut in N, then prove that val (f) cannot exceed C (P, $\overline{P}$). **(07 Marks)**

5 a. In a certain implementation of the programming language Pascal, an identifier consists of a single letter or a letter followed by upto seven symbols, which may be letters or digits. (26 letters, 10 digits). There are 36 reserved words. How many distinct identifiers are possible in this version of Pascal? **(07 Marks)**

b. How many bytes contain i) Exactly two 1's ; ii) Exactly four 1's ; iii) Exactly six 1's and iv) At least six 1's? **(07 Marks)**

c. In how many ways can 10 (identical) dimes be distributed among five children i) If there are no restrictions ; ii) Each child gets at least one dime ; iii) The oldest child gets at least 2 dimes. **(06 Marks)**

6 a. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3, 5. **(07 Marks)**

b. In how many ways can one arrange the letters in CORRESPONDENTS so that i) There is no pair of consecutive identical letters ; ii) There are exactly two pairs of consecutive identical letters. **(07 Marks)**

c. For the positive integers 1, 2, 3, 4, there are n derangements. Define derangements. What is the value of n? **(06 Marks)**

7 a. Give the generating function for :

i) 1, 1, 1, 1,........1, 1, 1.... all terms are 1

ii) 1, 1, 1, 1,........1, 0, 0, 0..... first terms are 1, others are 0

iii) 0, 1, 2, 3........ **(06 Marks)**

b. Find the generating function for $P_d(n)$, the number of partitions of a positive integer n into distinct summands. What is $P_d(6) = ?$ **(07 Marks)**

c. In each of the following, the function f(x) is the exponential generating function for the sequence $a_0, a_1, a_2$......., whereas the function g(x) is the exponential generating function for the sequence $b_0, b_1, b_2$....... Express g(x) in terms of f(x) if

i) $b_3 = 3$ and $b_n = a_n$, $n \in N, n \neq 3$.

ii) $b_1 = 2, b_2 = 4$, and $b_n = 2a_n$, $n \in N, n \neq 1, 2$. **(07 Marks)**

8 a. Solve the following recurrence relation :

$a_n = 5a_{n-1} + 6a_{n-2} \quad n \geq 2 \quad a_0 = 1, a_1 = 3.$ **(10 Marks)**

b. Solve the following recurrence relation:

$a_{n+1} - 2a_n = 2^n, \ n \geq 0 \quad a_0 = 1.$ **(10 Marks)**

c. Solve the following recurrence relation using the method of generating functions :

$a_{n+2} - 5a_{n+1} + 6a_n = 2, \quad n \geq 0, a_0 = 3, a_1 = 7.$

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