MATEE401

Reg. No. | | | | | | | | | | |

Fourth Semester B.E. Degree Examination, July/August 2002

Electrical Engineering

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: *i) Answer any FIVE full questions.*
ii) Usage of statistical Tables allowed.

1. (a) With usual notations, prove that

$$2nJ_n(x) = x[J_{n-1}(x) + J_{n+1}(x)]$$

(b) If $b > a$, prove that $\int_o^{\infty} Cos\ ax\ J_o(bx)dx = \frac{1}{\sqrt{b^2-a^2}}$

(c) Prove that $J_o^2(x) + 2\{J_1^2(x) + J_2^2(x) + J_3^2(x) + ...\} = 1$ **(6+7+7=20 Marks)**

2. (a) Show that $\int_{-1}^{1} x^2\ P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(2n+1)}{(2n-1)(2n+1)(2n+3)}$

(b) Prove that $(1-x^2)P_n^1(x) = (n+1)\{xP_n(x) - P_{n+1}(x)\}$

(c) Prove that $P_n(x) = \frac{1}{\pi} \int_0^{\pi} \frac{d\theta}{[x \pm \sqrt{x^2-1}Cos\theta]^{n+1}}$ **(7+6+7=20 Marks)**

3. (a) The first four moments of a frequency distribution about the point 5 are 2, 20, 40 and 50. Find $\bar{x}, \mu_2, \mu_3, \mu_4 \& \beta_2$.

(b) Find the first four moments about the mean for the following data.

x	12	14	16	18	20	22
f	1	4	6	10	7	2

(c) Show that the coefficient of Kurtosis is greater than 1. **(7+7+6=20 Marks)**

4. (a) Fit a curve of the form $y = ab^x$ for the following data.

x	1	2	3	4	5	6	7	8
y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

(b) If the tangent of the angle between the lines of regression of y on x & x on y is 0.6 and the standard deviation of y is twice the standard deviation of x, find the coefficient of correlation.

(c) Find the coefficient & correlation for the following data.

x	6	5	8	8	7	6	10	4	9	7
y	8	7	7	10	5	8	10	6	8	6

(7+6+7=20 Marks)

5. (a) A pair of dice is thrown. If it is known that one die shows a 4, what is the probability that

a) the other die shows a 5?

b) the total of both dice is greater than 7.

(b) A real estate man has 8 master keys to open several new homes. Only one master key will open any given house. If 40% of these homes are usually left

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unlocked, what is the probability that the real estate agent can get into a specific home if he selects 3 keys at random?

(c) Six coins are tossed. Find the probability of getting

i) exactly 3 heads.

ii) at least 3 heads.

iii) at least one head.

(6+7+7 Marks)

6. (a) Two random variables have the joint density given by

$$f(x,y) = \begin{cases} k(x^2+y^2), & 0 \le x \le 2, 1 \le y \le 4 \\ 0 & \text{elsewhere.} \end{cases}$$

Find $P(x+y>4)$.

(b) A random variable x has a mean $\mu = 10$, variance $\sigma^2 = 4$ and an unknown probability distribution. Find the constant C such that $P(\mid \times - 10 \mid \geq c) \leq 0.04$

(c) If x is a continuous random variable, having p.d.f defined by

$$f(x) = \begin{cases} 2e^{-zx}, & \text{x>0} \\ 0, & \text{elsewhere,} \end{cases}$$

find $P\{\mid \times \mid < 1/0.5 < x < 1.7\}$

(7+6+7=20 Marks)

7. (a) Find the mean and variance of Gamma distribution.

(b) The daily consumption of milk in a town, in excess of 30,000 litres follows Gamma distribution with $\alpha = 2$ and $\beta = 10,000$. The town has a daily stock of 40,000 litres. Find the probability that the stock will be adequate on a given day.

(c) $If\ f(x) = \frac{1}{\sigma\sqrt{2\pi}}\ exp\left\{\frac{-1}{2}\ \frac{(x-\mu)^2}{\sigma^2}\right\}, -\infty < x < \infty$. find the mean and the standard deviation of the random variable X.

(7+6+7=20 Marks)

8. (a) Explain the classification of optimization problems.

(b) Find all the basic solutions to the problem

Max Z = $x_1 + 3x_2 + 3x_3$

Subject to $x_1\ 2x_2\ 3x_3 = 4,$

$2x_1 + 3x_2 + 5x_3 = 7$

and $x_1, x_2, x_3 \geq 0$

Which of these solutions are

a) Basic feasible.

b) Optimal basic feasible.

(c) Solve the following linear programming problem using simplex method.

$$MaxZ = 3x_1 + 2x_2$$
$$x_1 + 2x_2 \leq 6$$
$$2x_1 + x_2 \leq 8$$
$$-x_1 + x_2 \leq 1$$
$$x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

(6+7+7=20 Marks)

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TE

MATL401

Reg. No. [ ]

Fourth Semester B.E. Degree Examination, January/February 2003

EC/TE/IT/BM/ML

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: ***Answer any FIVE full questions.***

1. (a) Find the solution of the Bessel's differential equation $x^2y'' + xy' + (x^2 - y^2)y = 0$ in the form $y = AJ_n(x) + BJ_{-n}(x)$ **(7 Marks)**

(b) Prove that $\frac{d}{dx}\{x^{-n}J_n(x)\} = x^{-n}J_{n+1}(x)$. **(6 Marks)**

(c) If α and β are the distinct roots of $J_n(x) = 0$, prove that

$$\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0.$$ **(7 Marks)**

2. (a) Establish Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$$ **(7 Marks)**

(b) Prove that $\int_{-1}^{+1} P_m(x)P_n(x)dx = \begin{cases} 0, & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$ **(6 Marks)**

(c) Express $x^4 - 2x^3 + 3x^2 - 4x + 5$ in terms of Legendre polynomial. **(7 Marks)**

3. (a) Use the graphical method to minimize z = 5x + 4y ; subject to the conditions
$x + 2y \geq 10; x + y \geq 8; 2x + y \geq 12$
$x \geq 0; y \geq 0$ **(7 Marks)**

(b) By graphical method maximize
$z = 3x + 4y$; subject to constraints
$2x + y \leq 40; \quad 2x + 5y \leq 180; \quad x \geq 0, y \geq 0$ **(6 Marks)**

(c) Solve the following L.P. problem, by simplex method
Maximize $z = x + 3y$
Subject *to* $x + 2y \leq 10$
$0 \leq x \leq 5, 0 \leq y \leq 4$ **(7 Marks)**

4. (a) Define the following terms
i) Slack and surplus variables
ii) Optimal solution
iii) Degeneracy in simplex method **(8 Marks)**

(b) Solve the following L.P.Problems by simplex method
Minimize $Z = x_1 - 3x_2 + 2x_3$
Subject to constraints
$3x_1 - x_2 + 2x_3 \leq 7$
$-2x_1 + 4x_2 \leq 12$
$-4x_1 + 3x_2 + 8x_3 \leq 10$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ **(12 Marks)**

5. (a) Show that the variance of first n-natural numbers is $\frac{n^2-1}{12}$. **(6 Marks)**

(b) Calculate the median, lower and upper quartiles for the following distribution.

Class	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	5	6	15	10	5	4	2	2

(7 Marks)

(c) The item of an observation are in A.P. a, a+d, a+2d, ... a+2nd. Find mean deviation from their mean. **(7 Marks)**

6. (a) If the event A and B are independent then show that $P(A \cup B) = P(A) + P(B)$. **(6 Marks)**

(b) A pair of dice is tossed twice. Find the probability of scoring 7 points

i) Once ii) at least once **(7 Marks)**

(c) There are three true coins and one false coin with head on both sides. A coin is chosen at random and tossed four times. If head occurs all the four times, what is the probability that the false coin has been chosen and used? **(7 Marks)**

7. (a) Find the mean and variance of the probability distribution by the following table

x_i	1	2	3	4	5
$P(x_i)$	0.2	0.35	0.25	0.15	0.05

(6 Marks)

(b) The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that

i) exactly two will be defective

ii) at least two will be defective

iii) none will be defective. **(7 Marks)**

(c) Find the expectation of the function $\phi(x) = xe^{-x}$ in a poisson distribution. **(7 Marks)**

8. (a) Define a stochastic process and classify the various types of stochastic process. **(6 Marks)**

(b) If Y and Z are two independent random variables with zero mean and equal standard deviation σ, find the mean and auto correlation of the process $\{x(t)\}$ where $x(t) = y + tz$ **(7 Marks)**

(c) Find the power spectrum of the random telegraph signal whose ACF is $(R)(\tau) = e^{-2\lambda|\tau|}; \lambda > 0$ **(7 Marks)**

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MATEE401

USN

Fourth Semester B.E Degree Examination, January/February 2004

Electrical & Electronics

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: *(i) Answer any FIVE full questions.*
(ii) All questions carry equal marks.

1. (a) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} sin\ x$ **(7 Marks)**

 (b) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$ **(7 Marks)**

 (c) Prove that $\int J_3(x)dx = C - J_2(x) - \frac{2}{x} J_1(x)$ **(6 Marks)**

2. (a) Prove that $P_n(x) = \frac{1}{n!\,2^n} \frac{d^n}{dx^n} (x^2-1)^n$ **(7 Marks)**

 (b) Prove that $(n+1)P_{n+1}(x) = x(2n+1)\ P_n(x) - nP_{n-1}(x)$ **(7 Marks)**

 (c) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomial. **(6 Marks)**

3. (a) Calculate the first four moments about the point $a = 4$ for the following distribution.

x:	0	1	2	3	4	5	6	7	8
f:	1	8	28	56	70	56	28	8	1

 Hence determine the first four moments. **(7 Marks)**

 (b) The first three moments of a distribution about the value 3 are 2, 10, −30. Show that the moments about x=0 are 5,31,141. Find the mean and variance. **(7 Marks)**

 (c) Compute the pearson's measure of skewness for the following distribution

Mid value (x) :	5	10	15	20	25	30	35	40
f:	2	108	580	175	80	32	18	5

(6 Marks)

4. (a) The two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$. Find

 i) mean of x's

 ii) mean of y's

 iii) the correlation coefficient between x and y **(7 Marks)**

 (b) If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $tan^{-1}(\frac{3}{8})$. Show that $\sigma_x = \frac{1}{2}\sigma_y$ **(7 Marks)**

 (c) By the method of least squares fit a straight line to the following data:

x:	1	2	3	4	5
y:	14	13	9	5	2

 Estimate the value of y when $x = 3.5$ **(6 Marks)**

5. (a) State and prove Baye's theorem. **(7 Marks)**

(b) A pair of dice is tossed twice. Find the probability of scoring 7 points

i) once ii) atleast once iii) twice **(7 Marks)**

(c) The probabibility density function of a variate X is

x:	0	1	2	3	4	5	6
P(X):	K	3K	5K	7K	9K	11K	13K

i) Find $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$

ii) What will be the minimum value of K so that $P(X \leq 2) > 3$? **(6 Marks)**

6. (a) Find the moment generating function of the exponential distribution $f(x) = \frac{1}{c}e^{\frac{-x}{c}}$, $0 \leq x \leq \infty$, $c > 0$. Hence find its mean and standard deviation. **(7 Marks)**

(b) The probability function $P(x)$ of a continuous random variable is given by

$$P(x) = y_0\, e^{-|x|}, -\infty < x < \infty$$

Prove that $y_0 = \frac{1}{2}$. Find the mean and variance of the distribution. **(7 Marks)**

(c) Define probability density function and cumulative distribution function. **(6 Marks)**

7. (a) Obtain the mean and variance of binomial distribution. **(7 Marks)**

(b) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain atleast 3 defective parts. **(7 Marks)**

(c) Fit a Poisson distribution to the set of observations.

x:	0	1	2	3	4
f:	122	60	15	2	1

(6 Marks)

8. (a) Define the terms: feasible solution, basic feasible solution, optimal solution and slack variables. **(7 Marks)**

(b) Use the simplex method to maximize $z = 2x + 4y$ subject to the constraints $3x + y \leq 22$, $2x + 3y \leq 24$, $x \geq 0$, $y \geq 0$ **(7 Marks)**

(c) Use the graphical method to minimize $z = 5x + 4y$ subject to the constraints:

$x + 2y \geq 10$, $x + y \geq 8$, $2x + y \geq 12$, $x \geq 0, y \geq 0$ **(6 Marks)**

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MATCH401

USN

Fourth Semester B.E. Degree Examination, January/February 2004

Chemical Engineering

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: ***1.Answer any FIVE questions.***
2.All questions carry equal marks.

1. (a) Fit a curve of the form $y = ab^x$ for the following data

x:	1	2	3	4	5	6	7	8
y:	1.0	1.2	1.8	2.5	3.6	4.7	6.6	9.1

(7 Marks)

(b) If $z = ax + by$ and r is the correlation coefficient between x and y, show that

$$r = \frac{\sigma_z^2 - (a^2\sigma_x^2 + b^2\sigma_y^2)}{2ab\sigma_x\sigma_y}$$

(6 Marks)

(c) Obtain the lines of regression and hence find the coefficient of correlation for the following data.

x:	1	3	4	2	5	8	9	10	13	15
y:	8	6	10	8	12	16	16	10	32	32

(7 Marks)

2. (a) Define i) Event
ii) Mutually exclusive events
iii) Independent events **(6 Marks)**

(b) If A and B are independent events, prove that $\overline{A}$ and $\overline{B}$ are also independent events. **(7 Marks)**

(c) The probability of conducting an examination on time is 0.95, if there is no strike by students and 0.25 if there is a strike. If the probability that there will be a strike is 0.65, find the probability of holding the examination on time. **(7 Marks)**

3. (a) A fair coin is tossed three times. Let X denote the number of heads showing up. Find the distribution of X. Also find its mean and variance. **(7 Marks)**

(b) Find the mean and standard deviations of the binomial distribution. **(7 Marks)**

(c) If X and Y are discrete random variables on a sample space S, prove that

$$COV(X,Y) = E[XY] - \mu_x\ \mu_y$$

(6 Marks)

4. (a) Show that $x^n J_n(x)$ is a solution of the differential equation

$$xy'' + (1-2n)y' + xy = 0$$

(6 Marks)

(b) Prove that $xJ_n'(x) = xJ_{n-1}(x) - nJ_n(x)$ **(7 Marks)**

(c) If n is an integer, prove that

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(xsin\theta - n\theta)d\theta$$

(7 Marks)

5. (a) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

(7 Marks)

(b) Show that

$$\frac{1+t}{(1-2xt+t^2)^{\frac{1}{2}}} = 1 + \sum_{n=0}^{\infty}[P_n(x) + P_{n+1}(x)]t^{n+1}$$

(7 Marks)

(c) By using Rodrigue's formula, prove the recurrence relation

$$P_n'(x) = xP_{n-1}'(x) + nP_{n-1}(x)$$

(6 Marks)

6. (a) Use the graphical method to minimize $z = 20x_1 + 10x_2$ subject to the constraints

$x_1 + 2x_2 \leq 40,$
$3x_1 + x_2 \geq 30,$
$4x_1 + 3x_2 \geq 60,$
$x_1 \geq 0, x_2 \geq 0$

(7 Marks)

(b) Use the simplex method to maximise $z = 3x + 4y$ subject to the constraints

$2x + y \leq 40,$
$2x + 5y \leq 180,$
$x \geq 0, y \geq 0$

(9 Marks)

(c) Define the artificial variables and slack variables.

(4 Marks)

7. (a) Use the simplex method to maximise $P = 60x + 50y$ subject to the constraints

$4x + 2y \leq 80,$
$3x + 2y \leq 60,$
$x \geq 0, y \geq 0$

(10 Marks)

(b) Minimize $z = 2x + 3y$ subject to the constraints

$x + y \geq 5, \quad x + 2y \geq 6; \quad x, y \geq 0$

(10 Marks)

8. (a) Using the bisection method find the root of the equation $xlog_{10}x = 1.2$ that lies between 2 and 3. Carry out five steps of approximations.

(7 Marks)

(b) By using the Secant method, find the real root of the equation $x^3 - 5x^2 - 29 = 0$ that lies between 5 and 6, correct to four decimal places.

(7 Marks)

(c) Show that the convergence in the Newton-Raphson's method is quadratil.

(6 Marks)

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MATL401

USN

Fourth Semester B.E. Degree Examination, January/February 2004

EC/TE/IT/BM/ML

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: ***1.Answer any FIVE full questions.***
2.All questions carry EQUAL marks.

1. (a) If α is a root of $J_n(ax) = 0$, prove that

$$\int_0^a x[J_n(\alpha x)]^2 dx = \frac{a^2}{2}\{J_n'(a\alpha)\}^2$$ (6 Marks)

(b) Prove that

i) $2nJ_n(x) = x[J_{n-1}(x) + J_{n+1}(x)]$

ii) $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$ (7 Marks)

(c) Starting from Jacobi's series, prove that

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(xsin\theta - n\theta)d\theta$$

hence deduce that

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} cos(xsin\theta)d\theta$$ (7 Marks)

2. (a) Prove that $\int_{-1}^{+1} P_n^2(x)dx = \frac{2}{2n+1}$ (6 Marks)

(b) Prove that

i) $(2n+1)xP_n(x) = (n+1)\ P_{n+1}(x) + nP_{n-1}(x)$

ii) $xP_n'(x) - P_{n-1}'(x) = nP_n(x)$ (7 Marks)

(c) If $v = (x^2 - 1)^n$, prove that $v_n = D^n v$ satisfies the Legendre's differential equation. Hence deduce the Rodrigue's formula for $P_n(x)$ (7 Marks)

3. (a) Define i) Feasible solution

ii) Optimal solution

iii) Slack and surplus variables (6 Marks)

(b) Maximize $z = x + 1.5y$ given $x \geq 0,\ y \geq 0$ subject to the constraints $x + 2y \leq 160,\ 3x + 2y \leq 240$ by graphical method. (7 Marks)

(c) Solve the following linear programming problem graphically:

Minimize $z = 20x + 10y$, subject to

$x + 2y \leq 40,$

$3x + y \geq 30,$

$4x + 3y \geq 60,$

$x \geq 0, y \geq 0$ (7 Marks)

4. (a) Explain degeneracy in Simplex method. (6 Marks)

(b) Use the simplex method to maximize $z = 3x + 4y$ subject to the constraints:

$2x + y \leq 40,$

$2x + 5y \leq 180,$

$x \geq 0, y \geq 0$ (7 Marks)

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MATC401

USN | | | | | | | | | | |

Fourth Semester B.E. Degree Examination, January/February 2004

(Civil, Transportation, Environmental and Ceramics)

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: ***1) Answer any FIVE full questions.***
2) All questions carry equal marks.
3)Statistical tables are allowed.

1. (a) Obtain $J_n(x)$ as a series solution of the Bessel's differential equation

$$x^2y'' + xy' + (x^2 - n^2)y = 0$$

(b) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$.

(c) Prove that

i) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$

ii) $J_{-n}(x) = (-1)^n.J_n(x)$ **(7+7+6 Marks)**

2. (a) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

(b) Prove that $\int_{-1}^{+1} P_m(x).P_n(x)dx = 0$ if $m \neq n$

(c) i) Prove that Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$$

ii) Find the value of $P_n(0)$. **(7+7+6 Marks)**

3. (a) Using graphical method, solve the following linear programming problem,

Maximize $z = x_1 - 3x_2$

Subject to the constraints

$$3x_1 + 4x_2 \geq 19$$
$$2x_1 - x_2 \leq 9$$
$$2x_1 + x_2 \leq 15$$
$$\text{and } x_1 - x_2 \geq 3$$
$$\text{where } x_1 \geq 0, x_2 \geq 0$$

(b) Solve the following L.P.P by Simplex method

Maximize $z = 3x + 4y$

Subject to the constraints

$$2x + y \leq 40$$
$$2x + 5y \leq 180$$

where $x \geq 0,\ y \geq 0$

(c) Define the following with illustrations.

i) Objective function ii) Feasible solution
iii) Optimisation iv) Slack and surplus variables

(7+7+6 Marks)

4. (a) Show that the Euler's equation

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

can be put in the following forms.

i) $\frac{d}{dx}\left[f - y'\frac{\partial f}{\partial y'}\right] - \frac{\partial f}{\partial x} = 0$

ii) $\frac{\partial f}{\partial y} - \frac{\partial^2 f}{\partial x \partial y'} - y'\frac{\partial^2 f}{\partial y \partial y'} - y''\frac{\partial^2 f}{\partial {y'}^2} = 0$

(b) Find the curve on which the functional $\int_0^1 [(y')^2 + 12xy]dx$ with $y(0) = 0$ and $y(1) = 1$ can be extremized.

(c) Find the extremal of the functional $\int_{x_1}^{x_2} (y^2 + {y'}^2 + 2ye^x)dx$ **(7+7+6 Marks)**

5. (a) Calculate mean deviation about the mean for the following distribution.

x :	3-4.9	5-6.9	7-8.9	9-10.9	11-12.9	13-14.9	15-16.9
f:	5	8	30	82	45	24	6

(b) The first three moments of a distribution about the value 2 of the variable are 1, 16 and -40. Show that the mean = 3, the variance = 15 and $\mu_3 = -86$

(c) Compute the quartile coefficient and skewness for the following distribution.

x:	1-10	11-20	21-30	31-40	41-50	51-60
f:	3	16	26	31	16	8

(7+7+6 Marks)

6. (a) Fit a straight line $y = a + bx$ to the following data by the method least squares.

x:	100	120	140	160	180	200
y:	45	55	60	70	80	85

(b) If θ is the angle between the two regression lines show that $tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x \, \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$.

Explain the significance when $r = 0$ and $r = \pm 1$.

(c) Compute the correlation coefficient and equations of the lines of regression for the data

x:	1	2	3	4	5	6	7
y:	9	8	10	12	11	13	14

(7 + 7 + 6 Marks)

7. (a) State and prove the multiplication law of probability.

(b) Fit a binomial distribution for the following distribution.

x:	0	1	2	3	4	5
f:	2	14	20	34	22	8

(c) If A and B are two events having $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$. Then compute

i) $P(A/B)$ ii) $P(B/A)$
iii) $P(\overline{A}/\overline{B})$ iv) $P(\overline{B}/\overline{A})$

(7+7+6 Marks)

8. (a) A set of five similar coins is tossed 320 times and the result is

No. of heads:	0	1	2	3	4	5
Frequency:	6	27	72	112	71	32

Calculate the value of χ^2

(b) A coin is tossed 144 times and a person gets 80 heads. Discuss whether the coin may be an unbiased one.

(c) Define the following terms.

i) Sampling distribution

ii) Level of significance

iii) Confidence limits.

(7+7+6 Marks)

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MATM401

USN

Fourth Semester B.E. Degree Examination, January/February 2004

(ME / AU / MI / IP / IM / MA)

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. All questions carry equal marks.
3. Graph sheets and statistical tables are allowed.

1. (a) Obtain $J_n(x)$ as a series solution of the differential equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ for $x \neq 0$ **(8 Marks)**

(b) Show that $xJ'_n(x) = nJ_n(x) - xJ_{n+1}(x)$ **(6 Marks)**

(c) If α and β are two distinct roots of $J_n(ax) = 0$ show that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ **(6 Marks)**

2. (a) State and prove the Rodrigues formula for $P_n(x)$ **(7 Marks)**

(b) Show that $\int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$ **(7 Marks)**

(c) Show that $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$ **(6 Marks)**

3. (a) Find the first four central moments of the following through the raw moments calculated about an assumed mean 67.

Class:	60 - 62	63 - 65	66 - 68	69 - 71	72 - 74
f:	5	18	42	27	8

(7 Marks)

(b) Find a second degree polynomial of the form $y = a + bx + cx^2$ that fits to the following data

x	0	1	2	3.	4
y	1	1.8	1.3	2.5	6.3

(7 Marks)

(c) Find the correlation coefficient and the regression lines for the following data

x	1	2	3	4	5
y	2	5	3	8	7

(6 Marks)

4. (a) A problem in mechanics is given to four students A, B, C and D; whose probabilities of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. What is the probability that the problem will be solved? **(6 Marks)**

(b) In a bolts factory, three machines A, B, C produce 25%, 35% and 40% of the total production. Of their product 5%, 4% and 2% are likely to be defective. If a bolt is drawn at random and is found to be defective, what is the probability that it was produced by machine 'B'. **(7 Marks)**

(c) In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of

Contd.... 2

10. Use Poisson distribution to calculate the approximate number of packets containing no defecctives, one defective and two defective blades respectively in a consignment of 10,000 packets. **(7 Marks)**

5. (a) The probability density function of a variate X is

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

Find $P[X < 4]$, $P[X \geq 5]$, $P[3 < X \leq 6]$ **(6 Marks)**

(b) Find the constant k such that $f(x) = \begin{cases} kx^2 & 0<x<3 \\ 0 & otherwise \end{cases}$ is a probability density function. Also compute

i) $P[1 < x < 2]$

ii) $P(X \leq 1)$

iii) $P[X > 1]$

iv) Mean

v) Variance **(7 Marks)**

(c) In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for

i) More than 2150 hours

ii) Less than 1950 hours

iii) More than 1920 but less than 2160 hours, given

$P[0 < z < 1.83] = 0.4664;$

$P[0 < z < 1.33] = 0.4082$ and

$P[0 < z < 2] = 0.4772$ **(7 Marks)**

6. (a) Explain the following terms:

i) Type I and Type II errors

ii) Null hypothesis

iii) confidence limits **(6 Marks)**

(b) A set of similar coins is tossed 320 times and the observations are:

No. of Heads:	0	1	2	3	4	5
Frequency:	6	27	72	112	71	32

Test the hypothesis that the data follows a binomial distribution. For 5 degrees of freedom, we have $\chi^2_{0.005} = 11.07$ **(7 Marks)**

(c) Find 95% confidence limits for the population mean given the following frequency distribution as a sample are

x	2	4	6	8	10
f	1	4	6	4	1

(7 Marks)

Assume the confidence coefficient to be 2.13.

Contd.... 3

7. (a) The nine items of a sample have the following values:

45, 47, 50 , 52, 48, 47, 49, 53, 51

Does the mean of these differ significantly from the assumed mean of 47.5. Given $t_{0.05} = 2.31$ for 8 degrees of freedom. **(7 Marks)**

(b) Find the mean time to failure for a Weibull distribution. **(7 Marks)**

(c) Explain the terms:

i) Reliability

ii) Failure density

iii) Exponential Failure law

iv) Meantime to failure

(6 Marks)

8. (a) Solve the following LPP using graphical method

Maximise $z = 5x + 3y$ subjected to the constraints

$4x + 5y \leq 1000$

$5x + 2y \leq 1000;$

$3x + 8y \leq 1200;$ **(7 Marks)**

$x \geq 0, y \geq 0$

(b) Explain the terms:

i) Slack variables and surplus variables

ii) Multiple optimal solutions

iii) unbounded solution

(6 Marks)

(c) Use the simplex method to

Maximise $z = 4x_1 + 3x_2 + 6x_3$

Subjected to $2x_1 + 3x_2 + 2x_3 \leq 440;$

$4x_1 + 3x_3 \leq 470;$

$2x_1 + 5x_2 \leq 430;$ **(7 Marks)**

$x_1 \geq 0;\ x_2 \leq 0;\ x_3 \geq 0$

** * **

MATL401

USN | | | | | | | | | | |

Fourth Semester B.E. Degree Examination, January/February 2005

EC/TE/IT/BM/ML

(Old Scheme)

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: ***1.Answer any FIVE full questions.***
2.All questions carry EQUAL marks.
3.Statistical tables are allowed.

1. (a) Prove that $xJ'_n(x) = nJ_n(x) - xJ_{n+1}(x)$

(b) Starting from the expression for $J_{\frac{1}{2}}(x)$ and $J_{-\frac{1}{2}}(x)$ in the standard form prove that

$$J'_{\frac{1}{2}}(x)J_{-\frac{1}{2}}(x) - J'_{\frac{-1}{2}}(x)J_{\frac{1}{2}}(x) = \frac{2}{\pi x}$$

(c) Show that

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} cos(xsin\theta)d\theta = \frac{1}{\pi}\int_0^{\pi} cos(xcos\theta)d\theta$$

(6+7+7 Marks)

2. (a) Obtain the solution of Lagendre Differential Equation.

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

(b) Prove that the generating function

$$(1-2xt+t^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} t^n P_n(x)$$

c) Evaluate i) $\int_{-1}^{+1} x^6 P_4(x)dx$

ii) $\int_{-1}^{+1} x^3 P_4(x)dx$

(6+6+8 Marks)

3. (a) Minimise $z = 5x + 4y$ subject to the constraints

$$x + 2y \geq 10, x + y \geq 8, 2x + y \geq 12, x \geq 0, y \geq 0$$

by Graphical Method.

(b) Explain the working procedure for simplex method.

(c) Use the simplex method to maximize $P = 60x + 50y$ subject to the constraints $4x + 2y \leq 80, 3x + 2y \leq 60, x \geq 0, y \geq 0$

(6+7+7 Marks)

4. (a) Maximise the function $P = 4x + 5y - 3z$ subject to the constraints $x + y + z = 10, x - y \geq 1, 2x + 3y + z \leq 40, x, y, z \geq 0$, by using big M-Method.

(b) Explain the following terms:

i) Difficulties in starting the simplex method

ii) Degeneracy in simplex method

iii) Graphical method

(8+12 Marks)

5. (a) Compute the Quartile deviation for the data

Class:	100-109	110-119	120-129	130-139	140-149	150-159	160-169	170-179
Frequency:	15	44	133	150	125	82	35	16

(b) The number of goals scored by two teams A & B in football season were as follows:

Goals scored:	0	1	2	3	4
No. of Matches: (team A)	27	9	8	5	4
No. of Matches: (team B)	17	9	6	5	3

Which team is more consistent?

(c) Compute the mean deviation and standard deviation for the data:

Size of the item:	6	7	8	9	10	11	12
Frequency:	3	6	9	13	8	5	4

(7 + 6 + 7 Marks)

6. (a) Prove that

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)+P(A\cap B\cap C)-P(A\cap B)-P(B\cap C)-P(C\cap A)$$

(b) Derive the expression for mean and standard deviation in the case of Poisson Distribution.

(c) Fit a Binomial distribution for the following data and find the Standard Deviation to compare with the theoretical standard deviation.

x:	0	1	2	3	4	5
f:	2	14	20	34	22	8

(6+7+7 Marks)

7. (a) In a test on 2000 electric bulbs it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and s.d of 60 hours. Estimate the number of bulbs likely to burn for

i) more than 2150 hours

ii) less than 1950 hours

(Given area at 1.83 = 0.4664 and area at 1.33 = 0.4082)

(b) A random variable has the following probability function:

x:	-2	-1	0	1	2	3
p(x):	0.1	k	0.2	$2k$	0.3	k

Find the value of k and calculate the value of mean and variance.

(c) Find k such that

$$f(x) = \begin{cases} kxe^{-x}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

is a p.d.f. Find the mean.

(8+6+6 Marks)

8. (a) Explain the classification of stochastic processes.

(b) Find the auto correlation $R(t_1, t_2)$ of the Stochastic Process defined by $x(t) = Acos(\omega t + \alpha)$, where the random variables A and α are independent and α is uniform in the interval $(-\pi, \pi)$.

(c) A Stochastic Process with its ensemble functions assumed to have equal probabilities are given by:

$$x_1(t) = 3, \quad x_2(t) = 3sin\ t, \quad x_3(t) = -3sin\ t$$
$$x_4(t) = 3cos\ t, x_5(t) = -3cos\ t, x_6(t) = -3$$

Show that the process is WSS but not SSS.

(6+8+6 Marks)

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NEW SCHEME

MAT41

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination, January/February 2005

Engineering Mathematics IV

Common to all branches

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*
2. All questions carry equal marks.
3. Use of statistical tables allowed.

Part A

1. (a) Derive Cauchy-Riemann equations in Cartesian form. **(7 Marks)**

(b) Find an analytic function $f(z) = u + iv$, given that

$u = x^2 - y^2 + \frac{x}{x^2+y^2}$ **(7 Marks)**

(c) Find the bilinear transformation that maps the points $z = -1, i, +1$ onto the points $w = 1, i, -1$ respectively. **(6 Marks)**

2. (a) State and prove Cauchy's integral formula. **(7 Marks)**

b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ as Laurent series in the regions

i) $1 < |z| < 2$ ii) $|z| > 2$ **(7 Marks)**

(c) Evaluate $\int_c \frac{e^{2z}}{(z+1)^3} dz$ where c is $|z| = \frac{3}{2}$ by using Cauchy's residue theorem. **(6 Marks)**

Part B

3. (a) If α is a root of $J_n(x) = 0$, prove that

$\int_0^1 x J_n^2(\alpha x) dx = \frac{1}{2}\{J_n'(\alpha)\}^2$ **(7 Marks)**

(b) Prove that

i) $2nJ_n(x) = x\{J_{n-1}(x) + J_{n+1}(x)\}$

ii) $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$ **(7 Marks)**

(c) Prove that $J_n(x) = \frac{1}{\pi}\int_0^\pi cos(n\theta - xsin\theta) d\theta$ where n is a positive integer. **(6 Marks)**

4. (a) If $v = (x^2 - 1)^n$, prove that $v_n = D^n v$ satisfies the Legendre's differential equation. Hence deduce the Rodrigue's formula for $P_n(x)$. **(7 Marks)**

(b) Prove that

i) $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$

ii) $xP_n'(x) - P_{n-1}'(x) = nP_n(x)$ **(7 Marks)**

(c) Prove that $\int_{-1}^{+1} P_n^2(x) dx = \frac{2}{2n+1}$ **(6 Marks)**

Contd.... 2

Part C

5. (a) Fit a parabola $y = a + bx + cx^2$ by the method of least squares to the following data:

x:	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y:	1.1	1.3	1.6	2.0	2.7	3.4	4.1

(7 Marks)

(b) In a partially destroyed laboratory record of correlation data, the following results only are available:

Variance of x is 9. Regression equations are $4x-5y+33 = 0, 20x-9y = 107$. Calculate

i) the mean values of x and y

ii) standard deviation of y, and

iii) the coefficient of correlation between x and y. **(7 Marks)**

(c) A box A contains 2 white and 4 black balls. Another box B contains 5 white and 7 black balls. A ball is transferred from box A to box B. Then a ball is drawn from box B. Find the probability that it is white. **(6 Marks)**

6. (a) The probability distribution of a random variable X is given by the following table:

x_i:	-2	-1	0	1	2	3
$P(x_i)$:	0.1	k	0.2	$2k$	0.3	k

i) Find the value of k and calculate the mean and variance.

ii) Find $P(X > -1)$ **(7 Marks)**

(b) Given that 2% of the fuses manufactured by a firm are defective, find, by using Poisson distribution, the probability that a box containing 200 fuses has

i) no defective fuses

ii) 3 or more defective fuses

iii) at least one defective fuse. **(7 Marks)**

(c) In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for

i) 10 minutes or more

ii) less than 10 minutes

iii) between 10 minutes and 12 minutes. **(6 Marks)**

Part D

7. (a) Explain the following terms:

i) Type I and Type II errors

ii) Null hypothesis

iii) Level of significance. **(7 Marks)**

(b) A die was thrown 1200 times and the number 6 was obtained 236 times. Can the die be considered fair at 0.01 level of significance? **(7 Marks)**

(c) A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure:5,2,8,-1,3,0,6,-2,1,5,0,4 (in appropriate units). Can it be concluded that, on the whole the stimulus will change the blood pressure? Use $t_{0.05}(11) = 2.201$. **(6 Marks)**

8. (a) The joint distribution of two random variables X and Y is given by the fc table:

$X \setminus Y$	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Determine the marginal distribution of X and Y. Also, verify that X and Y are independent. **(7 Marks)**

(b) A student's study habits are as follows: If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? **(7 Marks)**

(c) Define i) Transient state

ii) Recurrent state

iii) Absorbing state **(6 Marks)**

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MAT41

NEW SCHEME

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination, July/August 2005

Engineering Mathematics IV

Common to all branches

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*
2. Statistical tables are allowed.

PART - A

1. (a) If $f(z) = u + iv$ is analytic, Prove that **(7 Marks)**

$$\left[\frac{\partial}{\partial x} \mid f(z) \mid\right]^2 + \left[\frac{\partial}{\partial y} \mid f(z) \mid\right]^2 = \mid f'(z) \mid^2$$

(b) Give $u - v = (x - y)(x^2 + 4xy + y^2)$, find the analytic function $f(z) = u + iv$ **(7 Marks)**

(c) Find the bilinear transformation that maps the points $0, -i, -1$ of $Z-$ plane onto the points $i, 1, 0$ of W-plane respectively. **(6 Marks)**

2. (a) If a complex function $f(z)$ is analytic on and within a simple closed curve C then prove that $\oint_c f(z)dz = 0$ **(7 Marks)**

(b) Expand $f(z) = \frac{z}{(z-1)\,(z-3)}$ in power series valid for the regions i) $1 <\mid z \mid< 3$ ii) $\mid Z - 1 \mid< 2$ **(7 Marks)**

(c) Evaluate $\int_c \frac{Ze^z}{Z^2-1}\,dz$, where $C :\mid z \mid= 2$ **(6 Marks)**

PART - B

3. (a) Show that **(4+3 Marks)**

i) $2nJ_n(x) = x\,[J_{n+1}(x) + J_{n-1}(x)]$

ii) $\frac{d}{dx}\,[x^n J_n(x)] = x^n J_{n-1}(x)$

(b) Obtain the Jacobi's series **(7 Marks)**

$$cos(xsin\Phi) = J_0 + 2\,[J_2 cos2\Phi + J_4 cos4\Phi + ...]$$
$$sin(xsin\Phi) = 2J_1 sin\Phi + 2J_3 sin3\Phi + ...$$

(c) Prove that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(n\Phi - xsin\Phi)d\Phi$ where n is a positive integer. **(6 Marks)**

4. (a) Obtain the series solution of the Legendre's differential equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ **(7 Marks)**

Contd.... 2

(b) Establish the Rodrigue's formula

$$P_n(x) = \frac{1}{2^n \underline{|n}} \frac{d^n}{dx^n} (x^2 - 1)^n$$

(7 Marks)

(c) Show that $P_{2n}(0) = (-1)^n \frac{1.3.5...(2n-1)}{2.4.6...2n}$ and $P_{2n+1}(0) = 0$ **(6 Marks)**

PART - C

5. (a) Fit a curve of the form $y = a_0 + a_1 x + a_2 x^2$ to the data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

by the method of least squares. **(7 Marks)**

(b) In a partially destroyed lab record of analysis of correlation data, the following results are available. Variance of x is 9. Regression equations are $8x - 10y + 66 = 0$ and $40x - 18y - 214 = 0$.

Find $\overline{x}$, $\overline{y}, \sigma_y$ and correlation coefficient. **(7 Marks)**

(c) The probabilities that A, B, C hit a target are respectively $\frac{1}{6}$, $\frac{1}{4}$ and $\frac{1}{3}$. Each shoots once at the target. Find the probability that exactly one of them hits the target. If only one hits the targent, what is the probability that it was A. **(6 Marks)**

6. (a) The probability that a man aged 60 will live upto 70 is 0.65. Out of 10 men, now at the age of 60, find the probability that

i) at least 7 will live upto 70

ii) exactly 9 will live upto 70

iii) at most 9 will live upto 70 **(7 Marks)**

(b) A random variable X has the density function

$$p(x) = \begin{cases} kx^2 & for \ -3 \leq x \leq 3 \\ 0 & elsewhere \end{cases}$$

find k. Also find $P[x \leq 2]$ and $P[x > 1]$ **(7 Marks)**

(c) The life of an electric bulb is a normal variate with mean life of 2040 hours and standard deviation of 60 hours. Find the probability that a randomly selected bulb will burn for

i) more than 2150 hours

ii) less than 1950 hours

Given $P[0 \leq Z \leq 1.83] = 0.4664$; $P[0 \leq Z \leq 1.33] = 0.4082]$ **(6 Marks)**

PART - D

7. (a) Explain :

i) Null hypothesis

ii) Type I and Type II errors

iii) Significance level **(7 Marks)**

(b) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis, at 5% level of significance, that the coin is unbiased. **(7 Marks)**

(c) A sample of 12 measurements of the diameter of metal ball gave the mean $7.38mm$ with standard deviation $1.24m.m$ Find i) 95% and ii) 99% confidence limits for actual diameter. Given $t_{0.05}(11) = 2.20$ and $t_{0.01}(11) = 3.11$ **(6 Marks)**

8. (a) The joint probability distribution of two random variables X and Y is given below :

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find i) Marginal distributions of X and Y
ii) Covariance of X and Y **(7 Marks)**

(b) Three boys A, B and C are throwing a ball to each other. A always throws the ball to Band B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probability that A has the ball for the fourth throw. **(7 Marks)**

(c) Define :

i) Stochastic matrix

ii) Periodic state

iii) Absorbing state of a Markov chain **(6 Marks)**

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MATL401

Reg. No.

Fourth Semester B.E. Degree Examination, January/February 2006

EC/TE/IT/BM/ML

(Old Scheme)

Engineering Mathematics - IV

Time: 3 hrs.) (Max.Marks : 100

Note: ***Answer any FIVE full questions.***

1. (a) Show that $e^{\frac{x}{2}[t-\frac{1}{t}]} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$ **(7 Marks)**

   (b) Prove that **(4 Marks)**

   i) $2nJ_n(x) = x[J_{n+1}(x) + J_{n-1}(x)]$ **(3 Marks)**

   ii) $J_{-n}(x) = (-1)^n J_n(x)$, n is a positive integer.

   (c) Show that $\int_0^\infty e^{-ax} J_0(bx)dx = \frac{1}{\sqrt{a^2+b^2}}$, $a, b > 0$ **(6 Marks)**

2. (a) Prove that $P_n(x) = \frac{1}{2^n \underline{\lfloor n}} \frac{d^n}{dx^n}(x^2-1)^n$ **(7 Marks)**

   (b) Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials. **(7 Marks)**

   (c) Prove that $\int_{-1}^{+1} P_n^2(x)dx = \frac{2}{2n+1}$ **(6 Marks)**

3. (a) Define i) Optimal solution

   ii) Basic feasible solution **(7 Marks)**

   iii) Slack and surplus variables.

   (b) By using graphical method maximize $z = x - 3y, x, y \geq 0$ subject to the constraints.

   $3x + 4y \geq 19$

   $2x - y \leq 9$

   $2x + y \leq 15$ **(7 Marks)**

   $x - y \geq 3$

   (c) Using duality solve

   Minimize $Z = 0.7x_1 + 0.5x_2$

   Subject to $x_1 \geq 4,\ x_2 \geq 6$ **(6 Marks)**

   $x_1 + 2x_2 \geq 20$

   $2x_1 + x_2 \geq 18$

   $x_1, x_2 \geq 0$

Contd.... 2

4. (a) Find all the basic solutions of the following system of equations identifying in each case the basic and non-basic variables.

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

(6 Marks)

(b) Using simplex method to maximise

$z = 7x_1 + 12x_2 + 16x_3$, subject to the constraints

$$2x_1 + x_2 + x_3 \le 1$$
$$x_1 + 2x_2 + 4x_3 \le 2,\ x_1, x_2, x_3 \ge 0$$

(6 Marks)

(c) Using simplex method to maximise $z = 2x + y$

subject to the constraints

$$x + 4y \le 24$$
$$x + 2y \le 14$$
$$2x - y \le 8$$
$$x - y \ge 3,$$
$$x, y \ge 0$$

(8 Marks)

5. (a) The items of a certain observation are in A.P given by $a, a+d, a+2d, ...a+2nd$. Find the mean deviation from the mean. **(7 Marks)**

(b) Find the quartile deviation and standard deviation for the following data:

C.I	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
f	32	65	100	184	288	167	98	46	20

(7 Marks)

(c) A standard cell whose voltage is known to be 1.20 volts was used to test the accuracy of two voltmeters A and B. 10 independent readings of the voltage of the cell were taken and the result is as given below. Decide which of them is more reliable?

A	1.21	1.25	1.24	1.20	1.19	1.21	1.22	1.25	1.23	1.24
B	1.22	1.16	1.12	1.18	1.21	1.15	1.16	1.13	1.15	1.18

(6 Marks)

6. (a) State the axioms of probability.

Prove that $P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$ **(7 Marks)**

(b) State and prove BAYE's theorem. **(7 Marks)**

(c) A committee consists of 9 students two of which are from I year, three from 2nd year and four from 3rd year. Three students are to be removed at random. What is the chance that

i) the three students belong to different classes

ii) two belong to the same class and third to the different class

iii) the three belong to the same class. **(6 Marks)**

7. **(a)** The probability density function of a variate X is

X:	0	1	2	3	4	5	6
P(X):	k	3k	5k	7k	9k	11k	13k

Find k, mean and variance of the distribution. **(7 Marks)**

(b) Find the mean and S.D of the binomial distribution. **(7 Marks)**

(c) 2% of the fuses manufactured by a factory are found to be defective. Find the probability that a box containing 200 fuses contains

i) no defective fuses

ii) 3 or more defective fuses. **(6 Marks)**

8. **(a)** Define i) Stochastic process **(2 Marks)**

ii) Stationery process **(2 Marks)**

iii) Auto correlation **(3 Marks)**

(b) A random process $x(t)$ is represented by the ensemble

$$\left\{-k, -2k, -3k, k, 2k, 3k\right\}(k>0)$$

corresponding to the outcomes of an event which are equally probable. Show that the random process is stationery in the strict sense. **(7 Marks)**

(c) Define i) Ergodicity **(2 Marks)**

ii) Ergodicity mean **(2 Marks)**

iii) Wide sense stationery **(2 Marks)**

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 MATEE401

Reg. No. [| | | | | | | | | |]

Fourth Semester B.E Degree Examination, January/February 2006

Electrical & Electronics

(Old Scheme)

Engineering Mathematics - IV

Time: 3 hrs.) (Max.Marks : 100

Note: ***Answer any FIVE full questions.***
All questions carry equal marks.
Usage of statistical table is allowed.

1. **(a)** Define Bessel function $J_n(x)$. Using Bessel function prove that
$J'_n(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$ **(6 Marks)**

(b) Show that $\int_0^\infty e^{-ax} J_0(bx)dx = \frac{1}{\sqrt{a^2+b^2}}$ where $a, b > 0$ **(7 Marks)**

(c) Prove that $J_4(x) = \frac{8}{x}[\frac{6}{x^2} - 1]J_1(x) + (1 - \frac{24}{x^2})J_0(x)$ **(7 Marks)**

2. **(a)** Obtain the solution of legendre's differential equation
$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ **(6 Marks)**

(b) Prove that $P'_n(x) = xP'_{n-1}(x) + nP_{n-1}(x)$ **(7 Marks)**

(c) Express $x^3 + 2x^2 - 4x + 5$ interms of Legendre polynomials. **(7 Marks)**

3. **(a)** Compute the Pearson's coefficient of skewness for the data:

Marks above:	0	10	20	30	40	50	60	70	80	90
No.of Students:	140	134	122	101	78	55	30	16	5	1

(6 Marks)

(b) Compute the skewness based on third moment for the data:

Class:	0-2	2-4	4-6	6-8	8-10
Frequency:	5	18	42	27	8

(7 Marks)

(c) The first 3 moments of a distribution about the value 3 are 2, 10, -30. Show that the moments about $x = 0$ are 5, 31, 141. Find mean and variance. **(7 Marks)**

4. **(a)** Fit a straight line $y = ax + b$ to the following data:

x:	0	1	2	3	4	5
y:	9	8	24	28	26	20

(6 Marks)

(b) Obtain the lines of regression and hence find the coefficient of correlation for the data:

x:	1	2	3	4	5	6	7
y:	9	8	10	12	11	13	14

(7 Marks)

(c) The regression lines of y on x and x on y are $y = ax + b, x = cy + d$. Show that the ratio's of standard deviation of y on x is $\sqrt{a/c}$ and $\overline{x} = \frac{bc+d}{1-ac}, \overline{y} = \frac{ad+b}{1-ac}$ **(7 Marks)**

5. (a) Three machines A, B, C produce respectively $60\%, 30\%, 10\%$ of the total number of items of a factory. The percentages of defective output of these machines are respectively 2%, 3%, 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine 'C'. **(6 Marks)**

(b) Six coins are tossed. Find the probability of getting

i) exactly 3 heads

ii) at least 3 heads

iii) at least one head. **(7 Marks)**

(c) Define the axioms of probability. Then prove that

i) $P(\phi) = 0$ ii) $P(\overline{A}) = 1 - P(A)$ **(7 Marks)**

6. (a) If X and Y are random variables having joint density function

$$f(xy) = \begin{cases} 4xy & 0 \le x \le 1,\ 0 \le y \le 1 \\ = 0 & otherwise \end{cases}$$ verify that

i) $E(X+Y) = E(X) + E(Y)$ ii) $E(XY) = E(X)E(Y)$ **(6 Marks)**

(b) With the usual notation prove that mean = $\frac{1}{\alpha}$ and variance = $\frac{1}{\alpha^2}$ for an exponential distribution. **(7 Marks)**

(c) A uniform distribution has p.d.f.

$$f(x) = \begin{cases} \frac{1}{4}\ in\ 2 < x < 6 \\ 0 \quad otherwise \end{cases}$$

Find its mean and variance also find

i) $P(x \ge 1)$ ii) $P(x \le 4)$ iii) $P(x > 5)$ **(7 Marks)**

7. (a) With usual notation prove that $\overline{x} = np$ and $\sigma = \sqrt{npq}$ for a binomial distribution. **(6 Marks)**

(b) Fit a poisson distribution for the following data and calculate the theoretical frequencies

x:	0	1	2	3	4
f:	122	60	15	2	1

(7 Marks)

(c) Obtain the equation of the normal probability curve that may be fitted to the following data:

x:	6	7	8	9	10	11	12
f:	3	6	9	13	8	5	4

(7 Marks)

8. (a) Define feasible solution, optimal solution, slack variables and surplus variables. Write the standard form of L.P.P. **(6 Marks)**

Contd.... 3

(b) In the production of two types of watches a factory uses three machines A, B, C. The time for each watch on each machine and the maximum time available on each machine is given below.

Machine	Time Required		Maximum time available (in hrs)
	Watch I	Watch II	
A	6	8	380
B	8	4	300
C	12	4	404

The profit on Watch I is Rs.50 and on Watch II is Rs.30. Find what combination should be produced for the maximum profit. What is the maximum profit (use graphical method)? **(7 Marks)**

(c) Use the simplex method to minimize $P = x - 3y + 2z$ subject to the constraints

$3x - y + 2z \leq 7,$

$-2x + 4y \leq 12,$

$-4x + 3y + 8z \leq 10, x \geq 0, y \geq 0, z \geq 0$ **(7 Marks)**

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(b) In the production of two types of watches a factory uses three machines A, B, C. The time for each watch on each machine and the maximum time available on each machine is given below.

Machine	Time Required		Maximum time available (in hrs)
	Watch I	Watch II	
A	6	8	380
B	8	4	300
C	12	4	404

The profit on Watch I is Rs.50 and on Watch II is Rs.30. Find what combination should be produced for the maximum profit. What is the maximum profit (use graphical method)? **(7 Marks)**

(c) Use the simplex method to minimize $P = x - 3y + 2z$ subject to the constraints

$3x - y + 2z \leq 7,$

$-2x + 4y \leq 12,$

$-4x + 3y + 8z \leq 10, x \geq 0, y \geq 0, z \geq 0$ **(7 Marks)**

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MATCH401

Reg. No.

Fourth Semester B.E. Degree Examination, January/February 2006

Chemical Engineering

(Old Scheme)

Engineering Mathematics - IV

Time: 3 hrs.) (Max.Marks : 100

Note: ***Answer any FIVE full questions.***

1. (a) Derive : $\sigma^2_{ax+by} = a^2\sigma_x^2 + b^2\sigma_y^2 + 2r\ ab\sigma_x\ \sigma_y$. Also compute the correlation coefficient r, σ_x and σ_y given that $\sigma^2_{x+y} = 15, \sigma^2_{x-y} = 11,\ \sigma^2_{2x+y} = 29$

   (b) If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $tan^{-1}(3/5)$ show that $\sigma_y = 2\sigma_x$ and $\sigma_x = 2\sigma_y$

   (c) Fit a second degree parabola by the method of least squares for the following data

x:	1	2	3	4	5
y:	10	12	13	16	19

(7+7+6 Marks)

2. (a) If A and B are two events having $P(A) = \frac{1}{2},\ \ P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. Compute

   i) $P(A/B)$ ii) $P(\overline{A}/\overline{B})$

   (b) Obtain the Poisson distribution as the limiting form of the binomial distribution.

   (c) 4 coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and also test the goodness of fit using chi-square test given that $\chi^2_{0.05} = 9.49$ for 4d.f

No. of Heads	0	1	2	3	4
Frequency	5	29	36	25	5

(6+7+7 Marks)

3. (a) Explain bisection method and use it to find a real root of the equation $cosx - 1.3x = 0$ correct to 5 decimal places (perform 4 iterations)

   (b) Use the regula - falsi method to find the third approximation to the root of the equation $tanx + tanhx = 0$ lying between 2 and 3 correct to 4 decimal places (perform 3 iterations).

   (c) Use Newton-Raphson method to derive an iterative formula for finding k^{th} root of N and hence find fourth root of 22.

(6+7+7 Marks)

Contd.... 2

4. **(a)** Starting from the definition of $J_n(x)$ prove that:

$$J_{n+1}(x) + J_{n-1}(x) = \frac{2n}{x} J_n(x)$$

Hence show that

$$J_4(x) = \frac{8}{x}\left(\frac{6}{x^2} - 1\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$$

(b) Show that $\int_0^\infty e^{-ax} J_0(bx)dx = \frac{1}{\sqrt{a^2+b^2}}, a, b > 0.$

(c) Show that $J_0^2 + \sum_{k=1}^{\infty} 2J_k^2 = 1$ **(6+7+7 Marks)**

5. **(a)** Solve $\nabla^2 f = 0$ in spherical system leading to Legendre's differential equation.

(b) Show that $P'_{n+1}(x) - P'_{n-1}(x) = (2n+1)P_n(x)$

(c) Expand the function $f(x) = \begin{cases} 0 \ in \ -1 < x < 0 \\ x \ in \ 0 < x < 1 \end{cases}$ in terms of Legendre polynomials. **(7+7+6 Marks)**

6. **(a)** Form the P.D.E by eliminating the arbitrary functions from the relation.

$$z = xf_1(x+t) + f_2(x+t)$$

(b) Solve $\frac{\partial^3 z}{\partial x^3} + 4\frac{\partial z}{\partial x} = 0$, given that $z = 0, \frac{\partial z}{\partial x} = 0, \frac{\partial^2 z}{\partial x^2} = 4$ when $x = 0$

(c) Solve by the method of separation of variables

$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ where $u(0, y) = 2e^{5y}$ **(7+7+6 Marks)**

7. **(a)** Derive the D'Alembert's solution of the wave equation $u_{tt} = C^2 u_{xx}$ subject to the conditions: $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t} = 0$ when $t = 0$

(b) A string of length l is tightly stretched between the points $x = 0$ and $x = l$. From this position it is given an initial velocity $b sin^3(\pi x/l)$. Find the displacement at any time t. **(10+10 Marks)**

8. **(a)** Obtain the various possible solutions of the one dimensional heat equation $u_t = C^2 u_{xx}$ by the method of separation of variables. Which solution is conformal for solving a physical problem?

(b) Solve the heat equation $u_t = C^2 u_{xx}$ subject to the conditions $u(0, t) = 0,\ u(\pi, t) = 0,\ u(x, 0) = \pi x - x^2$ in $(0, \pi)$ **(10+10 Marks)**

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Reg. No. | | | | | | | | | |

Fourth Semester B.E. Degree Examination, January/February 2006

(Civil, Transportation, Environmental and Ceramics)

(Old Scheme)

Engineering Mathematics - IV

Time: 3 hrs.) (Max.Marks : 100

Note: *1) Answer any FIVE full questions.*
2) All questions carry equal marks.
3)Statistical tables are allowed.

1. (a) Obtain the expressions for $J_{1/2}(x)$ and $J_{-1/2}(x)$

   (b) Prove that $x\ J_n^1(x) = n\ J_n(x) - x\ J_{n+1}(x)$

   (c) Starting from the generating function for $J_n(x)$ obtain Jacobi series.

(6+7+7=20 Marks)

2. (a) Prove the recurrence relation :

$$x\ P_n^1(x) - P_{n-1}^1(x) = nP_n(x)$$

   (b) Find $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$, and express $f(x) = 2x^3 - x^2 - 3x + 2$ interms of Legendre polynomials.

   (c) Prove that $\int_{-1}^{1} P_m(x) \cdot P_n(x)\ dx = 0\ \ for\ \ m \neq n$

(6+7+7 Marks)

3. (a) Define the terms :
   i) Objective function
   ii) Decision variables
   iii) Standard form of L.P.P

   (b) Use the graphical method to maximize $z = 5z_1 + 3x_2$

   Subject to

$$4x_1 + 5x_2 \leq 1000$$
$$5x_1 + 2x_2 \leq 1000$$
$$3x_1 + 8x_2 \leq 1200$$
$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

   (c) Solve the following L.P. problem by the simplex method

   Minimize $P = x - 3y + 2z$

   Subject to

$$3x - y + 2z \leq 7$$
$$-2x + 4y \leq 12$$
$$-4x + 3y + 8z \leq 10$$
$$x \geq 0,\ y \geq 0,\ z \geq 0$$

(6+7+7 Marl

Contd..

4. (a) Prove that if $I = \int_{x_1}^{x_2} f(x, y, y')dx$, then

$$\delta \int_{x_1}^{x_2} f(x, y, y')dx = \int_{x_1}^{x_2} \delta f(x, y, y')dx$$

(b) Solve the variational problem :

$\delta \int_0^1 (x + y + y'^2)dx = 0$ under the conditions $y(0) = 1, \ y(1) = 2$

(c) Prove that the geodesics in a plane are straight lines.

(6+7+7 Marks)

5. (a) The items of an observation are in the A.P. $a, \ a+d, \ a+2d, - - - - a+2nd.$ Find the standard deviation.

(b) Define skewness and find the quartile coefficient of skewness for the following data :

Marks :	55-58	58-61	61-64	64-67	67-70
No. of students :	12	17	23	18	11

(c) The first four moments of a distribution about the value 5 of the variable are 2, 20, 40 and 64. Calculate the mean, variance, coefficient of skewness and coefficient of kurtosis.

(6+7+7 Marks)

6. (a) Given :

	x-series	y-series
Mean	18	100
S.D.	14	20

and $r = 0.8$. Write down the equations of lines of regression and find the most probable value of y when $x = 70$.

(b) Fit a second degree parabola for the data in the form $y = ax^2 + bx + c$ by the method of least squares.

x :	0	1	2	3	4
y :	1	1.8	1.3	2.5	2.3

(c) Find the coefficient of correlation between the industrial production and export using the following data and comment on the result.

Production (in crore tons) :	55	56	58	59	60	60	62
Export (in crore tons) :	35	38	38	39	44	43	44

(6+7+7 Marks)

7. (a) State and prove addition rule of probability for any 2 events A and B.

(b) A and B throw alternately with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his chance of winning.

(c) In a certain college, 4% of boys and 1% of girls are taller than 1.8m. Furthermore 60% of the students are girls. If a student is selected at random and is found taller than 1.8m, what is the probability that the student is a girl? **(6+7+7 Marks)**

8. **(a)** Define the following :

i) Sampling distribution

ii) Standard error

iii) Null hypothesis.

(b) Prove that the mean and variance are equal in the Poisson's distribution.

(c) A set of 5 similar coins is tossed 320 times and result is :

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. **(6+7+7 Marks)**

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NEW SCHEME

MAT41

Reg. No.

Fourth Semester B.E Degree Examination, January/February 2006

Common to all branches

Engineering Mathematics IV

Time: 3 hrs.) (Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*
2. All questions carry equal marks.
3. Use of statistical tables allowed.

Part A

1. (a) Define the derivative of a complex function. Obtain Cauchy-Riemann equations in polar form as a set of necessary conditions for $f(Z)$ to be analytic. **(7 Marks)**

   (b) Determine the analytic function whose real point is $y + e^x \cos y$. **(7 Marks)**

   (c) Find the Bi-linear transformation that maps $z = 1,\ i,\ -1$ onto the points $2,\ i,\ -2$ respectively. **(6 Marks)**

2. (a) State and prove Cauchy's theorem. **(7 Marks)**

   (b) Expand $\frac{1}{(z-1)(z-2)}$ in a series of powers of z that is valid in the following regions.

   i) $|z| < 1$ ii) $1 < |z| < 2$ iii) $|z| > 2$ **(7 Marks)**

   (c) Use Cauchy's residue theorem to find

   $$\int_c \frac{e^z}{z^2+4}\, dz$$

   where c is the circle $|z - i| = 2$. **(6 Marks)**

Part - B

3. (a) Obtain the series solution of Bessel's equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ in the form $y = AJ_n(x) + BJ_{-n}(x)$. **(7 Marks)**

   (b) Show that

   i) $J_{-n}(x) = (-1)^n J_n(x), \quad n = 1,2,3...$

   ii) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x.$ **(7 Marks)**

   (c) Derive Bessel's integral formula

   $$J_n(x) = \frac{1}{\pi} \int_0^{\pi} \cos\,(n\theta - x \sin\theta)d\theta$$

   where n is a positive integer. **(6 Marks)**

Contd....

NEW SCHEME

MAT41

Reg. No. ☐☐☐☐☐☐☐☐☐☐

Fourth Semester B.E Degree Examination, January/February 2006

Common to all branches

Engineering Mathematics IV

Time: 3 hrs.) (Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*
2. All questions carry equal marks.
3. Use of statistical tables allowed.

Part A

1. (a) Define the derivative of a complex function. Obtain Cauchy-Riemann equations in polar form as a set of necessary conditions for $f(Z)$ to be analytic. **(7 Marks)**

   (b) Determine the analytic function whose real point is $y + e^x \cos y$. **(7 Marks)**

   (c) Find the Bi-linear transformation that maps $z = 1,\ i,\ -1$ onto the points $2,\ i,\ -2$ respectively. **(6 Marks)**

2. (a) State and prove Cauchy's theorem. **(7 Marks)**

   (b) Expand $\frac{1}{(z-1)(z-2)}$ in a series of powers of z that is valid in the following regions. **(7 Marks)**

   i) $|z| < 1$ ii) $1 < |z| < 2$ iii) $|z| > 2$

   (c) Use Cauchy's residue theorem to find

   $$\int_c \frac{e^z}{z^2+4}\, dz$$ **(6 Marks)**

   where c is the circle $|z - i| = 2$.

Part - B

3. (a) Obtain the series solution of Bessel's equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ in the form $y = AJ_n(x) + BJ_{-n}(x)$. **(7 Marks)**

   (b) Show that **(7 Marks)**

   i) $J_{-n}(x) = (-1)^n J_n(x), \quad n = 1,2,3...$

   ii) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\, \sin x.$

   (c) Derive Bessel's integral formula

   $$J_n(x) = \frac{1}{\pi} \int_0^{\pi} \cos\,(n\theta - x \sin\theta) d\theta$$ **(6 Marks)**

   where n is a positive integer.

Contd.... 2

4. (a) Prove the Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$$

(7 Marks)

(b) Show that

$$(1 - 2xz + z^2)^{\frac{-1}{2}} = \sum_{n=0}^{\infty} P_n(x) \cdot z^n$$

(7 Marks)

(c) Establish the orthogonality property

$$\int_{-1}^{1} P_m(x) P_n(x) dx = 0, \quad m \pm n$$

where m, n are positive integers.

(6 Marks)

Part - C

5. (a) Using the method of least squares, find the parabola $y = a + bx + cx^2$ which fits the data

x	−3	−2	−1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

(7 Marks)

(b) The lines of regression of y on x and x on y are given as

$4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate means of x and y and coefficient of correlation between them.

(7 Marks)

(c) State and prove Baye's theorem.

(6 Marks)

6. (a) Find the mean and standard deviation of the Binomial distribution. (7 Marks)

(b) Find the constant C such that

$$f(x) = \begin{cases} Cx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

is a probability density function. Also compute $P(1 < x < 2)$. (7 Marks)

(c) In a test on 2000 electric bulbs it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for more than 1920 hours but less than 2160 hours.

(6 Marks)

Part - D

7. (a) The mean life time of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the life time of all the bulbs produced by the company, test the hypothesis $\mu = 1600$ hours against the alternative hypothesis $\mu \neq 1600$ hours using a level of significance of 0.01.

(7 Marks)

Contd.... 3

(b) A random sample of 10 boys had the following I.Q.

70, 120, 110, 101, 88, 83, 95, 98, 107, 100

Do these data support the assumption of a population mean I.Q. of 100 at 5% level of significance? **(7 Marks)**

(c) A die is tossed 960 times and it falls with 5 upwards 184 times. Is the die biased at 5% level of significance? **(6 Marks)**

8. (a) The joint probability distribution of two discrete random variables x and y is given by the table. Determine the marginal distributions of x and y. Also find whether x and y are independent.

x \ y	1	3	6
1	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{18}$
3	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$
6	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{36}$

(7 Marks)

(b) Find the unique fixed probability vector of the following regular stochastic matrix. **(7 Marks)**

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

(c) Bob's study habits are as follows : If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is only 60% sure not to study the next night as well. Find out how often, in the long run, Bob studies. **(6 Marks)**

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MATL-401

USN | | | | | | | | | | |

OLD SCHEME

Fourth Semester B.E. Degree Examination, July 2006
Common to E&C,TC,IT,BME

Engineering Mathematics-IV

Time : 3 hrs.] [Max. Marks : 100

Note : *1. Answer any FIVE questions.*
2. Statistical Tables are allowed.

1 a. Obtain series solution of the Bessel differential equation.
$x^2 y^{11} + xy^1 + (x^2 - n^2) y = 0, (x \neq 0)$. **(08 Marks)**

b. If n is an integer, prove that $J_n(-x) = (-1)^n J_n(x) = J_{-n}(x)$. **(06 Marks)**

c. Prove that:
$J_0^2(x) = 2[J_1^2(x) + J_2^2(x) + J_3^2(x) + \ldots\ldots\ldots.] = 1.$ **(06 Marks)**

2 a. Express the poly nomial $f(x) = 2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(05 Marks)**

b. Prove that:
i) $(2n+1)\, xp_n(x) = (n+1)p_{n+1}(x) + np_{n-1}(x)$
ii) $np_n(x) = x\, p_n^1(x) - p_{n-1}^1(x)$ **(08 Marks)**

c. Show that:
$$\int_{-1}^{1} x^2\, p_{n+1}(x)\, p_{n-1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$ **(07 Marks)**

3 a. Use the graphical method to minimize z= 20x + 10y subject to the constraints
$x + 2y \leq 40$, $3x + y \geq 30$, $4x + 3y \geq 60$, where $x \geq 0$, $y \geq 0$. **(10 Marks)**

b. Solve the following LP problem by the simplex method:
Maximize $\phi = x_1 + 3x_2$ subject to $x_1 + 2x_2 \leq 10$ where $o \leq x_1 \leq 5$, $0 \leq x_2 \leq 4$ **(10 Marks)**

4 a. In the production of two types of watches a factory uses three machines A, B, C. The time required for each watch on each machine and the maximum time available on each machine is give below:

Machine	Time required		Maximum time available (in hours)
	Watch I	Watch II	
A	6	8	380
B	8	4	300
C	12	4	404

The profit on Watch I is Rs.50 and on Watch II is Rs.30. Find what combination should be produced for the maximum profit. What is the maximum profit? **(08 Marks)**

b. Maximize z = 2x+y subject to the constraints $x+4y \leq 24$, $x+2y \leq 14$, $2x-y \leq 8$, $x-y \leq -3$; $x \geq 0$, $y \geq 0$. **(12 Marks)**

Contd....2

5 a. Find α from the following frequency distribution table if the mean is 25, and then find the mediun and the mode.

Class:	0-10	10-20	20-30	30-40	40-50
Frequency:	14	25	27	α	15

(07 Marks)

b. In a set of 150 observations, the mean and the standard deviation were quoted as 120 and 15 respectively. However, it was discovered that one of the observations was taken as 15 instead of 105. Find the correct mean and the correct standard deviation. **(07 Marks)**

c. Show that in a frequency distribution, the mean deviation from the mean is less than the standard deviation. **(06 Marks)**

6 a. Let A and B be the events with $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$. Find i) $P(A \cup B)$ ii) $P(\overline{A} \cap \overline{B})$ iii) $P(\overline{A} \cup \overline{B})$ iv) $P(A \cap \overline{B})$ v) $P(B \cap \overline{A})$.. **(05 Marks)**

b. Let A and B be independent events. Show that i) A and $\overline{B}$, and ii) $\overline{A}$ and B are independent. If A and B are not mutually exclusive, show that $\overline{A}$ and $\overline{B}$ are independent. **(07 Marks)**

c. Let S be a finite sample space and the events $A_1, A_2, \ldots\ldots A_n$ form a partition of S. For any event E, prove that $P\left(\frac{Ai}{E}\right) = \frac{P\left(\frac{E}{Ai}\right)P(Ai)}{\sum P\left(\frac{E}{Ai}\right)P(Ai)}$ where $i=1, 2, 3, \ldots n$ **(08 Marks)**

7 a. Define a binomial distribution. Find its mean and variance. **(08 Marks)**

b. Given that 2% of the fuses manufactured by a firm are defective, find the probability that a box containing 200 fuses has

i) Atleast one defective fuse

ii) 3 or more defective fuses. **(04 Marks)**

c. Define movement generating function $M_x(t)$ for discrete and continuous random variables. Prove the following relations:

i) $M_x(t) = 1 + \mu_1^1 t + \mu_2^1 \frac{t^2}{\angle 2} + \ldots\ldots + \mu_r^1 \frac{t^r}{\angle r} + \ldots\ldots$ where $\mu_1^1, \mu_2^1 \ldots\ldots \mu_r^1 \ldots\ldots$ are the moments about origin.

ii) $M_{x+y}(t) = M_x(t)\, M_y(t)$ given that x and y are independent random variables. **(08 Marks)**

8 a. Consider a stochastic process $\{x(t)\}$ defined on a finite sample space $S = \{S_1, S_2, S_3\}$ by $x(t,s_1) = 5$, $x(t,s_2) = 5\cos 2t$, $x(t,s_3) = 5\sin 2t$. Let $P(s_1) = P(s_2) = P(s_3) = \frac{1}{3}$. Find the mean, variance and ACF for the process. **(07 Marks)**

b. Prove for a random signal process, the mean is Zero and the autocorrelation and auto covariance are both equal to $e^{-2\lambda|t_1 - t_2|}$ in the usual form. **(08 Marks)**

c. Let $\{X(t)\}$ and $\{Y(t)\}$ be two mean–ergodic processes with means μ_1 and μ_2 respectively, and $Z(t) = X(t) + AY(t)$ where A is a random variable independent of Y(t) and taking values 0 and 1 with equal probability. Prove that the process $\{Z(t)\}$ is not mean ergodic. **(05 Marks)**

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MAT41

USN

NEW SCHEME

Fourth Semester B.E. Degree Examination, July 2006
Common to All Branches
Engineering Mathematics - IV

Time: 3 hrs.] [Max. Marks:100

Note: *1. Answer any FIVE full questions choosing at least ONE question from each part.*
2. Use of Statistical tables permitted.

Part - A

1 a. If f(z) = u + iv where u and v are functions of x, y is an analytic function, show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$. **(07 Marks)**

b. Find the analytic function, whose real part is $x\sin x\cosh y - y\cos x\sinh y$. **(07 Marks)**

c. Find the bilinear transformation that maps z = 1, i, -1 onto the points ω = 0, 1, ∞. **(06 Marks)**

2 a. State and prove Cauchy's theorem. **(07 Marks)**

b. Expand $f(z) = \frac{z}{(z-1)(z-3)}$ in a series of powers of z that is valid in the regions i) $1 < |z| < 3$ ii) $|z-1| < 2$ **(07 Marks)**

c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where $C : |z| = 3$ **(06 Marks)**

Part – B

3 a. If 'α' is a root of $J_n(x) = 0$, prove that $\int_0^1 x J_n^2(\alpha x)dx = \frac{1}{2}[J_n'(\alpha)]^2$. **(07 Marks)**

b. Show that $2nJ_n(x) = x[J_{n-1}(x) + J_{n+1}(x)]$ and hence show that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x} - \cos x\right]$. **(07 Marks)**

c. Establish the Jacobi's series
$\cos(x\sin\theta) = J_0 + 2[J_2 \cos 2\theta + J_4 \cos 4\theta + ...]$
$\sin(x\sin\theta) = 2[J_1 \sin\theta + J_3 \sin 3\theta + ...]$ **(06 Marks)**

4 a. Obtain the series solution of the Legendre's differential equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$. **(07 Marks)**

b. Show that $\int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$. **(07 Marks)**

c. Express $f(x) = x^3 + 2x^2 - x - 3$ interms of Legendre polynomials. **(06 Marks)**

Contd.... 2

Part – C

5 a. Find a second degree polynomial that fits to the following data : **(07 Marks)**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

b. Find the correlation coefficient and the regression lines of y on x and x on y for the following data : **(07 Marks)**

x	1	2	3	4	5
y	2	5	3	8	7

c. In a factory, machines A, B, C produce respectively 25%, 35% and 40% of the total production of their output 5%, 4% and 2% are defective. An item is drawn from the factory and is found be defective. Find the probability that it was manufactured by A. **(06 Marks)**

6 a. The probability distribution of a finite random variable X is given by the following table :

x_i	:	-2	-1	0	1	2	3
$P(x_i)$	:	0.1	k	0.2	2k	0.3	k

i) Find the value of k and calculate the mean and variance.
ii) Evaluate $P(X<1)$. **(07 Marks)**

b. Find the mean and variance of the Binomial distribution. **(07 Marks)**

c. The length of a telephone conversation is an exponential variate with mean 3 minutes. Find the probability that a call i) ends in less than 3 minutes ii) takes between 3 to 5 minutes. **(06 Marks)**

Part – D

7 a. The weights of workers in a large factory are normally distributed with mean 68 kgs and standard deviation 3 kgs. If 80 samples consisting of 35 workers each are chosen, how many of the 80 samples will have the mean between 67 and 68.25 kgs? Given $P[0 \le z \le 2] = 0.4772$ and $P[0 \le z \le 0.5] = 0.1915$. **(07 Marks)**

b. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level significance. **(07 Marks)**

c. Explain the following terms :
i) Type I and Type II errors. ii) Null hypothesis. **(06 Marks)**

8 a. A Joint probability distribution is given by,

x \ y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find covariance of (X, Y) and correlation coefficient ρ of (X, Y). **(07 Marks)**

b. Find the fixed probability vector of the following regular stochastic matrix :

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$ **(07 Marks)**

c. A student's study habits are as follows :
If he studies one night, he is 60% sure not to study the next night; on the other hand if he does not study one night, he is 80% sure to study the next night. In the long run how often does he study? **(06 Marks)**

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MATL401

USN | | | | | | | | | |

1998 SCHEME

Fourth Semester B.E. Degree Examination, Dec.06 / Jan.07
EC / TE / IT / BM / ML

Engineering Mathematics - IV

Time: 3 hrs.] [Max. Marks:100

Note: ***1. Answer any FIVE full questions.***
2. Use of Statistical tables is allowed.

1 a. If n is a positive integer prove that,
$J_{-n}(x) = (-1)^n J_n^-(x)$ **(06 Marks)**

b. Prove that
i) $2J_n^{'}(x) = J_{n-1}(x) - J_{n+1}(x)$
ii) $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$ **(07 Marks)**

c. Prove that generating function,
$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$. **(07 Marks)**

2 a. Obtain the series solution of the Legendre' equation,
$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$ **(08 Marks)**

b. Express $f(x) = x^3 - 5x^2 + x + 2$ interms of Legendre's polynomial. **(06 Marks)**

c. Show that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$ if $m \neq n$ **(06 Marks)**

3 a. Find all basic solutions and the optimal basic solution for the following problem,
Maximize $\phi = 2x_1 + 3x_2 + 4x_3 + 7x_4$
Subject to $2x_1 + 3x_2 - x_3 + 4x_4 = 8$,
$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$
$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0$ **(08 Marks)**

b. Use the graphical method to
Maximize $Z = 10x_1 + 20x_2$
Subject to the constraints
$5x_1 + 2x_2 \leq 20$,
$3x_1 + 4x_2 \leq 30$,
and $x_1 \geq 0,\ x_2 \geq 0$. **(06 Marks)**

c. Use the graphical method to
Minimize $Z = 20x_1 + 10x_2$
Subject to the constraints
$x_1 + 2x_2 \leq 40$,
$3x_1 + x_2 \geq 30$,
$4x_1 + 3x_2 \geq 60$,
$x_1 \geq 0,\ x_2 \geq 0$. **(06 Marks)**

Contd....2

4 a. Use simplex method

Maximize $Z = x + 1.5y$

Subject to the constraints

$x + 2y \le 160$,

$3x + 2y \le 240$,

$x \ge 0,\ y \ge 0$. **(10 Marks)**

b. Use the simplex method to

Maximize $P = 7x + 12y + 16z$

Subject to the constraints

$2x + y + z \le 1$,

$x + 2y + 4z \le 2$,

$x, y, z \ge 0$. **(10 Marks)**

5 a. The mean of 200 items was 50. Later on it was discovered that two items were measured as 92 and 8 instead of 192 and 88. Find out the correct mean. **(06 Marks)**

b. Compute the mean deviation from the average for the following data : **(07 Marks)**

Class	140-150	150-160	160-170	170-180	180-190	190-200
f	4	6	10	18	9	3

c. Find the mean and S.D of the first 'n' natural. **(07 Marks)**

6 a. State the axioms of probability. Prove that

i) $P(\overline{A}) = 1 - P(A)$

ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

where A and B are any two events. **(07 Marks)**

b. A problem in mathematics is given to four students. The probabilities of their solving the problem are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively. Find the probability that the problem will be solved. **(07 Marks)**

c. State and prove Baye's theorem of probability.

$$P\left(\frac{A_i}{A}\right) = \frac{P(A_i)P(A/A_i)}{\sum_{i=1}^{n} P(A_i)P\left(\frac{A}{A_i}\right)}.$$ **(06 Marks)**

7 a. Find the value of K such that the following distribution represents a finite probability distribution. Hence find its mean and standard deviation also find $P(x \le 1) P(x > 1)$ and $P(-1 < x \le 2)$. **(06 Marks)**

x	-3	-2	-1	0	1	2	3
P(x)	k	$2k$	$3k$	$4k$	$3k$	$2k$	k

b. A certain screw making machine produces on an average two defectives out of 100 and packs them in boxes of 500. Find the probability that a box containing 15 defectives. **(07 Marks)**

c. Obtain the mean and variance of binomial distribution. **(07 Marks)**

8 a. Define Auto correlation, Auto covariance, Correlation coefficient. **(06 Marks)**

b. Define Ergodicty and obtain its mean. **(07 Marks)**

c. Find the auto correlation $R(t_1,\ t_2)$ of the stochastic process defined by $X(t) = A\cos(\omega t + \alpha)$ where the random variables A and α are independent and α is uniform in the interval [-π, π]. **(07 Marks)**

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MAT 41

USN | | | | | | | | | |

NEW SCHEME

Fourth Semester B.E. Degree Examination, Dec.06/Jan. 07
Common For All Branches
Engineering Mathematics - IV

Time: 3 hrs.] [Max. Marks:100

Note: ***1. Answer any FIVE questions choosing at least ONE from each part.***
2. Statistical tables are permitted.

PART A

1 a. Find an analytic function f(z)=u+iv, where $u-v = (x-y)(x^2+4xy+y^2)$. **(06 Marks)**

b. If $u = \frac{x^2}{y}, y \neq 0$ and $V = x^2 + 2y^2$, show that the curves u = constant and v = constant are orthogonal but f(z) = u+iv is not an analytic function. **(07 Marks)**

c. Find the bilinear Transformation which maps the points 1, I, -1 onto the points 2, I, -2 respectively. **(07 Marks)**

2 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Expand $f(z) = \frac{z}{(z-1)(z-3)}$ in a Laurent's series valid for the regions i) $|z| < 1$ ii) $1 < |z| < 3$ iii) $|z-1| < 2$. **(07 Marks)**

c. Using Cauchy's residue theorem evaluate

$$\int_c \frac{z\operatorname{Cos}z}{(z-\pi/2)^3}\,dz,$$ where C is the circle $|z-1| = 1$. **(07 Marks)**

PART B

3 a. With usual notations prove that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$ **(06 Marks)**

b. i) Prove that $J'_n(x) = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$

ii) Prove that $J_n(-x) = (-1)^n J_n(x)$. **(07 Marks)**

c. prove that if α and β are two distinct roots of $J_n(x) = 0$ then

$$\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = 0 \quad (\alpha \neq \beta)$$ **(07 Marks)**

4 a. Prove that $P_n(x) = \frac{1}{\angle n 2^n} \frac{d^n}{dx^n}\left[(x^2-1)^n\right]$ where n is a positive integer. **(06 Marks)**

b. Express the functions $f(x) = x^3+2x^2-x-3$ in terms of Legendre's polynomials. **(07 Marks)**

c. Prove that $\int_{-1}^{+1} P_n^2(x)\,dx = \frac{2}{2n+1}$ **(07 Marks)**

Contd... 2

PART C

5 a. Fit a parabola of the form $y = a + bx + cx^2$ for the following data. **(06 Marks)**

X	-2	-1	0	1	2
Y	-3.150	-1.390	0.620	2.880	5.378

b. If θ is the acute angle between the two regression lines relating the variables x and y, show that $\tan\theta = \frac{(1-r^2)}{r}\frac{\sigma_x\sigma_y}{\sigma_x^{\ 2}+\sigma_y^{\ 2}}$. **(07 Marks)**

c. In a certain college, 4% of men students and 1% of women students are taller than 1.8m. Further more, 60% of the students are women. If a student is selected at random and is found taller than 1.8m, what is the probability that the student is a woman? **(07 Marks)**

6 a. Find the mean and variance of the probability distribution given by the following table **(06 Marks)**

Xi	0	1	2	3	4
P(Xi)	0.2	0.35	0.25	0.15	0.05

b. Obtain the mean and variance of the Binomial distribution. **(07 Marks)**

c. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for i) less than 10 minutes ii) 10 minutes or more? **(07 Marks)**

PART D

7 a. Explain the terms :
i) Type I and Type II errors ii) Level of significance. **(06 Marks)**

b. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of confidence. **(07 Marks)**

c. A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 (in appropriate units). Can it be concluded that, on the whole, the stimulus will change the blood pressure. Use $t_{0.05}(11) = 0.201$. **(07 Marks)**

8 a. Verify that $f(x, y) = \begin{cases} e^{-(x+y)} & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$ is a density function of a Joint probability distribution. Also evaluate $P(x < 1)$. **(06 Marks)**

b. Prove that the Markov chain whose transition probability matrix is $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible. Find the corresponding stationary probability vector. **(07 Marks)**

c. A student's study habits are as follows. If he studies at night, he is 70% sure not to study the next night. On the other hand if he does not study at night, he is 60% sure not to study the next night. In the long run how often does he study? **(07 Marks)**

MATL401

USN

OLD SCHEME

Fourth Semester B.E. Degree Examination, July 2007
Engineering Mathematics IV

Time: 3 hrs.] [Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. Obtain $J_n(x)$ as a series solution of the Bessel's differential equation:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$$ **(07 Marks)**

b. Starting from Jacobi's series, prove that

$$J_n(x)=\frac{1}{\pi}\int_0^{\pi}\cos(x\sin\theta-n\theta)d\theta$$

where n is an integer. **(07 Marks)**

c. Prove that

i) $J_{5/2}(x)=\sqrt{\frac{2}{\pi x}}\left[\left(\frac{3-x^2}{x^2}\right)\sin x-\left(\frac{3}{x}\right)\cos x\right]$ and

ii) $J_{-5/2}(x)=\sqrt{\frac{2}{\pi x}}\left[\left(\frac{3}{x}\right)\sin x-\left(\frac{3-x^2}{x^2}\right)\cos x\right]$. **(06 Marks)**

2 a. Establish Rodrigue's formula for Legendre's polynomials viz.,

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$ **(07 Marks)**

b. With usual notation, prove that

$$\int_{-1}^{1}P_m(x)P_n(x)dx=0 \qquad (m\neq n)$$ **(07 Marks)**

c. Express x^3+2x^2-4x+5 in terms of Legendre's polynomials. **(06 Marks)**

3 a. A firm manufactures three products A, B and C. The profits are Rs.3, Rs.2 and Rs.4 respectively. The firm has two machines M_1 and M_2 and below is the required capacity processing time in minutes for each machine on each product.

Machine	Product		
	A	B	C
M_1	4	3	5
M_2	2	2	4

Machines M_1 and M_2 have 2000 and 2500 machine minutes respectively. The firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's. Set up an L.P.P to maximize profit. **(10 Marks)**

3 b. Use graphical method to maximize $Z = 5x_1 + 3x_2$

Subject to the conditions $4x_1 + 5x_2 \leq 1000$;

$5x_1 + 2x_2 \leq 1000$;

$3x_1 + 8x_2 \leq 1200$;

$x_1 \geq 0;\ x_2 \geq 0$. **(10 Marks)**

4 a. Define the following terms:

i) Canonical and standard forms of L.P.P.

ii) Slack and surplus variables.

iii) Feasible solution and optimal solution. **(10 Marks)**

b. Using simplex method,

maximize $Z = x + 3y$

Subject to the constraints $x + 2y \leq 10$;

$0 \leq x \leq 5$;

$0 \leq y \leq 4$. **(10 Marks)**

5 a. Define

i) Standard deviation.

ii) Root-mean-square (RMS) deviation.

With usual notation, prove that $s^2 = \sigma^2 + d^2$. **(07 Marks)**

b. Find the median and quartile deviation for the following data: **(07 Marks)**

Class:	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency:	14	17	22	26	23	18

c. Find the variance of first n-natural numbers. **(06 Marks)**

6 a. Define axiomatic probability. If A and B are any two arbitrary events then prove that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(07 Marks)**

b. An office has four secretaries handling respectively 20%, 60%, 15% and 5% of the files of all government reports. The probability that they misfile such reports are respectively 0.05, 0.1, 0.1 and 0.05. Find the probability that the misfiled report can be blamed on the first secretary. **(07 Marks)**

c. A four figure number is formed of the figures 1, 2, 3, 5 without repetition. Find the chance that the number is divisible by 5. **(06 Marks)**

7 a. For Poisson's distribution, show that mean and variance are equal. **(07 Marks)**

b. When a coin is tossed four times, find the probability of getting

i) One head

ii) At least three heads.

iii) At most three heads. **(07 Marks)**

c. Define discrete and continuous random variables with one example each. **(06 Marks)**

8 a. Find the average autocorrelation function of the sine wave function $f(t) = A\sin(\omega_1 t + \phi)$ where $\omega_1 = {2\pi}/{T_i}$. **(07 Marks)**

b. Find the power spectrum of the random telegraph signal whose ACF is $R(\tau) = e^{-2\lambda|\tau|}, \lambda > 0$. **(07 Marks)**

c. Define a stochastic process and classify the various types of stochastic process. **(06 Marks)**

USN

NEW SCHEME

Fourth Semester B.E. Degree Examination, July 2007
Common to All Branches
Engineering Mathematics – IV

Time: 3 hrs.] [Max. Marks:100

Note : *1. Answer any FIVE full questions, choosing atleast one question from each part.*
2. Use of statistical tables allowed.

PART A

1 a. Define an analytic function and obtain Cauchy-Reimann equations in the Cartesian form. **(07 Marks)**

b. Find the analytic function f(Z) whose imaginary part is $e^x(x\sin y + y\cos y)$. **(07 Marks)**

c. Discuss the transformation $\omega = e^Z$. **(06 Marks)**

2 a. Derive the Cauchy's integral formula $f(a) = \frac{1}{2\pi i}\int_c \frac{f(Z)}{Z-a}dZ$. **(07 Marks)**

b. Expand $f(Z) = \left(\frac{Z-1}{Z+1}\right)$ in Taylor's series about the points i) Z = 0; ii) Z = 1. **(07 Marks)**

c. Determine the poles of the function $f(Z) = \frac{Z^2}{(Z-1)^2(Z+2)}$ and the residue at each pole. **(06 Marks)**

PART B

3 a. Obtain the series solution of the Bessel's differential equation in the form: $y = AJ_n(x) + BJ_{-n}(x)$. **(07 Marks)**

b. Prove that $J_0^{\,2} + 2J_1^{\,2} + 2J_2^{\,2} + 2J_3^{\,2} + ------ = 1$. **(07 Marks)**

c. Prove that:

i) $2nJ_n(x) = x[J_{n+1}(x) + J_{n-1}(x)]$

ii) $J_n{}'(x) = \frac{1}{2}[J_{n+1}(x) + J_{n-1}(x)]$. **(06 Marks)**

4 a. Prove that: $(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x).t^n$ **(07 Marks)**

b. Prove that: $\int_{-1}^{+1} P_m(x).P_n(x)dx = 0$ if $m \neq n$. **(07 Marks)**

c. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(06 Marks)**

Contd.... 2

PART C

5 a. Fit a curve of the form $y = ab^x$ by using the least square method to the following data: **(07 Marks)**

x :	1	2	3	4	5	6	7
y :	87	97	113	129	202	195	193

b. Obtain the lines of regression and hence find the coefficient of correlation for the following data: **(07 Marks)**

x :	1	3	4	2	5	8	9	10	13	15
y :	8	6	10	8	12	16	16	10	32	32

c. State and prove Baye's theorem. **(06 Marks)**

6 a. Define the random variables and classify. Find the value of K such that the following distribution represents a finite probability distribution. Hence find its mean and standard deviation. Also find i) $P(x \le 1)$ ii) $P(x > 1)$ iii) $P(-1 < x \le 2)$. **(07 Marks)**

x :	-3	-2	-1	0	1	2	3
P(x) :	K	2K	3K	4K	3K	2K	K

b. Find the mean and variance of the binomial distribution. **(07 Marks)**

c. 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains i) no defective fuse ii) 3 or more defective fuses. **(06 Marks)**

PART D

7 a. Explain the following terms:
i) Type I and Type II errors
ii) Null hypothesis
iii) Level of significance. **(07 Marks)**

b. In 324 throws of a six faced die, an odd number turned up 181 times. Is it reasonable to think that the die is an unbiased one? **(07 Marks)**

c. Find the student's 't' for the following variable values in a sample of eight: -4, -2, -2, 0, 2, 2, 3, 3, taking the mean of the universe to be zero. **(06 Marks)**

8 a. The joint distribution of two random variables X and Y is as follows:

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Compute the following:
i) E(X) and E(Y) ii) E(XY) iii) COV(X, Y). **(07 Marks)**

b. Define stochastic matrix. Find the unique fixed probability vector for the regular stochastic matrix.

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$$

(07 Marks)

c. Explain: i) Transient state ii) Recurrent state iii) Absorbing state of a Markov chain. **(06 Marks)**

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USN | | | | | | | | | | |

MAT41

Fourth Semester B.E. Degree Examination, Dec. 07 / Jan. 08

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note : ***1. Answer any FIVE full questions, choosing at least one question from each part.***
2. Use of statistical tables is permitted.

PART A

1 a. If $f(z)=u(x,y)+i\,v(x,y)$ is analytic show that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2=4|f'(z)|^2$. **(07 Marks)**

b. Find the analytic function whose real part is $e^x[(x^2-y^2)\text{Cos}y-2xy\text{Sin}y]$. **(07 Marks)**

c. Find the bilinear transformation that maps the points 0, -i, -1 of Z-plane onto the points i, 1, 0 of W-plane respectively. **(06 Marks)**

2 a. Evaluate $\int_C \frac{\text{Sin}\pi z^2+\text{Cos}\pi z^2}{(z-1)(z-2)}dz$, where C is the circle $|z|=3$, using Cauchy's integral formula. **(07 Marks)**

b. Expand $f(z)=\frac{z}{(z-1)(z-3)}$ in power series valid for the regions:

i) $1<|z|<3$ ii) $0<|z-1|<2$. **(07 Marks)**

c. Find the poles and the residue at each pole for the function:

$f(z)=\frac{z^2}{(z-1)^2(z+2)}$. **(06 Marks)**

PART B

3 a. Obtain a series solution of the Bessel's differential equation:

$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$ leading to $J_n(x)$. **(07 Marks)**

b. Show that: i) $2nJ_n(x)=x[J_{n+1}(x)-J_{n-1}(x)]$ ii) $\frac{d}{dx}[x^nJ_n(x)]=x^nJ_{n+1}(x)$. **(07 Marks)**

c. Show that $J_0^2+2J_1^2+2J_2^2+-----+2J_n^2=1$. **(06 Marks)**

4 a. Establish the Rodrigue's formula for Legendre polynomials. **(07 Marks)**

b. Show that $\int_{-1}^{1}P_n^2(x)dx=\frac{2}{2n+1}$. **(07 Marks)**

c. Show that i) $P_n(1)=1$ and ii) $P_n(-1)=(-1)^n$. **(06 Marks)**

PART C

5 a. Fit a parabola $y=a+bx+cx^2$ to the data:

x	1	2	3	4
y	1.7	1.8	2.3	3.2

by the method of least squares. **(07 Marks)**

b. Find the correlation coefficient and the two regression lines for the following data:

x	1	2	3	4	5
y	2	5	3	8	7

Find the best estimate for y when x = 3.5 and the best estimate for x when y = 3.5. **(07 Marks)**

c. In a factory, machines A, B, C respectively produce 25%, 35% and 40% of the total production. 5%, 3% and 2% of their outputs are defective. If an item selected at random is found to be defective, find the probability that it was produced by machine A. **(06 Marks)**

6 a. Find the mean and variance of a binomial distribution. **(07 Marks)**

b. A certain screw making machine produces on an average two defectives out of 100 and packs them in boxes of 500. Find the probability that the box contains:
i) Three defectives ii) Atleast one defective iii) Between two and four defectives. **(07 Marks)**

c. In a test on 2000 electric bulbs, it was found that the life of a bulb is a normal variate with average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for i) More than 2150 hours ii) Less than 1950 hours.
Given that $P[0 \le Z \le 1.83] = 0.4664$ and $P[0 \le Z \le 1.33] = 0.4082$. **(06 Marks)**

PART D

7 a. Explain the following terms:
i) Type I and Types II errors
ii) Null hypothesis
iii) Level of significance. **(07 Marks)**

b. A random sample of 100 recorded deaths in past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does the data indicate that the average life span today is greater than 70 years? Use a 0.05 level of significance. **(07 Marks)**

c. A machinist is making engine parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with a standard deviation of 0.04 inch. On the basis of this sample, would you say that the work is inferior? Test the hypothesis at 5% level of significance. Given $t_{0.05}(\gamma = 9)$ is 2.262. **(06 Marks)**

8 a. A fair coin is tossed three times. Let X denote 0 or 1 according as a head or tail occurs on the first toss and let Y denote the number of heads which occur. Find: i) The distributions of X and Y ii) Joint distribution of X and Y iii) COV(X, Y). **(07 Marks)**

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probabilities that i) A has the ball ii) B has the ball and iii) C has the ball, for the fourth throw. **(07 Marks)**

c. Explain: i) Transient state ii) Recurrent state iii) Absorbing state of a Markov chain. **(06 Marks)**

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MATL401

Fourth Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note : *1. Answer any FIVE full questions.*
2. Graph sheets and statistical tables are allowed.

1 a. With usual notations, prove that
$xJ_n'(x) = nJ_n(x) - x.J_{n+1}(x)$ **(07 Marks)**

b. Show that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x) = -J_1(x).t^n$ **(07 Marks)**

c. Prove that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$, and hence deduce that $J_0'(x) = -J_1(x)$. **(06 Marks)**

2 a. Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials. **(06 Marks)**

b. Prove that $\int_{-1}^{1} P_{m(x)}.P_n(x).dx = 0$ if $m \neq n$

$= \frac{2}{(2n+1)}$ if $m \neq n$ **(07 Marks)**

c. Prove that
i) $(2n + 1)x. P_n(x) = (n + 1) P_{n+1}(x) + n. P_{n-1}(x)$
ii) $P_n(1) = 1$ **(07 Marks)**

3 a. Minimize Z=5x + 4y subject to the constraints $x + 2y \geq 10$, $x + y \geq 8$, $2x + y \geq 12$, $x \geq 0$, $y \geq 0$ by graphical method. **(10 Marks)**

b. Use simplex method to maximize $z = x + 1.5y$ subject to the constraints $x + 2y \leq 160$, $3x + 2y \leq 240$, $x \geq 0$, $y \geq 0$. **(10 Marks)**

4 a. Solve the problem :
Maximize $\phi = 2x_1 + 3x_2 + 5x_3$ subject to $x_1 - 2x_3 = 0$
$x_2 + x_3 = 1$
$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$ **(07 Marks)**

b. Define the terms :
i) Objective function ii) Feasible solution iii) Slack and surplus variable. **(06 Marks)**

c. Find all the basic solutions of the following system of equations identifying in each case the basic and non – basic variables.
$2x_1 + x_2 + 4x_3 = 11$
$3x_1 + x_2 + 5x_3 = 14.$ **(07 Marks)**

5 a. Find the mean, median and mode for the following frequency distribution.

Class	0 – 7	7 – 14	14 – 21	21 – 28	28 – 35	35 – 42	42 - 49
Frequency	19	25	36	72	51	43	28

(06 Marks)

b. Show that in a frequency distribution, the mean deviation from the mean is less than the standard deviation. **(07 Marks)**

c. The mean of five items of an observation is 4 and the variance is 5.2. If the three of five items are 1, 2 and 6, find the other two. **(07 Marks)**

6 a. If two events A and B in a finite sample space "S" are mutually independent but not mutually exclusive, prove that their compliments $\overline{A}$ and $\overline{B}$ in S are mutually independent. **(06 Marks)**

b. The probability that a contractor will get a plumbing contract is 2/3 and the probability he will not get an electric contract is 5/9. If the probability of getting at least one contract is 4/5. What is the probability that he will get both contracts? **(07 Marks)**

c. Three players A, B, C of a cricket team try for the captaincy and their chances of getting it are in the proportion 4 : 2 : 3 respectively. The probability that A, if he is made the captain will drop a particular player 'D' from the team is 0.3. The probability that B and C doing the same are 0.5 and 0.8 respectively. What is the probability that 'D' will not be dropped from the team? **(07 Marks)**

7 a. Find the value "K" such that the following distribution represents a finite probability distribution. Hence find its Mean and standard deviation.

X	-3	-2	-1	0	1	2	3
P(x)	K	2K	3K	4K	3K	2K	K

(07 Marks)

b. The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured then find the probability that :
i) Exactly two are defective
ii) At least 2 pens are defective. **(06 Marks)**

c. Derive expressions for Mean and standard deviation in the case of Poisson distribution. **(07 Marks)**

8 a. Define a stochastic process and classify various types of stochastic processes. **(06 Marks)**

b. Find auto correlation, Auto covariance and correlation co-efficient of X(t) with reference to the random process.

Outcomes	1	2	3	4	5	6
X(t)	-2	-1	1	2	-t	t

(07 Marks)

c. A stochastic process is defined on a finite sample space with three sample points. Its description is provided by the specifications of the three sample functions
$X(t, \lambda_1) = 3$, $X(t, \lambda_2) = 3 \cos t$, $X(t, \lambda_3) = 3 \operatorname{Sin} t$
and the probability of the assignment is
$P(\lambda_1) = P(\lambda_2) = P(\lambda_3) = \frac{1}{3}$
Compute $\mu(t)$ and $R(t_1, t_2)$
Decide whether the process is SSS or WSS. **(07 Marks)**

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Fourth Semester B.E. Degree Examination, Dec.08 / Jan.09

Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note : ***Answer FIVE full questions choosing atleast two from each part.***

Part A

1 a. Find by Taylor's series method the value of y at $x = 0.1$ and $x = 0.2$ to five places of decimals from $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ consider upto 4^{th} degree terms. **(06 Marks)**

b. Apply Runge-Kutta method to find an approximate value of y for $x = 0.2$ in steps of 0.1 of $\frac{dy}{dx} = x + y^2$, given that y = 1, when x = 0. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1+y)$ and y(1)=1, y(1.1)=1.233, y(1.2)=1.548, y(1.3)=1.979, evaluate y(1.4) by Adam's-Bashforth method. **(07 Marks)**

2 a. Derive Cauchy – Riemann equations in polar-form. **(06 Marks)**

b. Determine the analytic function, $f(z) = u + iv$, if
$$u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}.$$ **(07 Marks)**

c. Discuss the transformation $w = e^z$. **(07 Marks)**

3 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Find the Taylor's expansion of $f(z) = \frac{2z^3 + 1}{z^2 + z}$ about the point z = i. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$. **(07 Marks)**

4 a. Solve in series the equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$. **(06 Marks)**

b. Reduce the differential equation $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + k^2xy = 0$ to Bessel's equation. **(07 Marks)**

c. Derive the Rodrigue's formula, $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2 - 1)^n$. **(07 Marks)**

Part B

5 a. Fit a second degree polynomial to the following data: **(06 Marks)**

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

b. The two regression equations of the variables x and y are
$x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$
Find i) mean of x's ii) mean of y's and iii) the correlation coefficient of x and y. **(07 Marks)**

c. State and prove Baye's theorem. **(07 Marks)**

6 a. The probability density function of a variate x is

x	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

i) Find k.

ii) Find P(x<4), and $P(3 < x \le 6)$. **(06 Marks)**

b. Derive mean and variance for the Poisson distribution. **(07 Marks)**

c. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. if 60 hours. Estimate the number of bulbs likely to burn for

i) More than 2150 hours

ii) Less than 1950 hours and

iii) More than 1920 hours, but less than 2160 hours. **(07 Marks)**

7 a. In a city A, 20% of a random sample of 900 school boys has a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? **(06 Marks)**

b. A machinist is making engine parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with a S.D. of 0.04 inch. On the basis of this sample, would you say that the axle is inferior? **(07 Marks)**

c. A set of five similar coins is tossed 320 times and the result is:

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. **(07 Marks)**

8 a. The joint distribution of two random variables x and y is given by the following table:

x \ y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Determine the marginal distribution of x and y. Also verify that x and y are stochastically independent. **(06 Marks)**

b. Find the fixed probability vector of the regular stochastic matrix,

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}.$$

(07 Marks)

c. Explain i) Transient state ii) Recurrent state iii) absorbing state of Markov chain. **(07 Marks)**

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Fourth Semester B.E. Degree Examination, June-July 2009
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Solve $\frac{dy}{dx} = 2y + 3e^{\lambda}$, $y(0) = 0$. Using Taylor's series method an find y(0.1), y(0.2). **(06 Marks)**

b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ at x = 0.2 with step length h = 0.2. **(07 Marks)**

c. Use Milne's predictor –corrector method to find y at x = 0.8, given $\frac{dy}{dx} = x - y^2$ with,

X	0	0.2	0.4	0.6
Y	0	0.02	0.0795	0.1762

Apply corrector once. **(07 Marks)**

2 a. Find the analytic function f(2) = u + iv if $v = e^x (x \sin y + y \cos y)$. **(06 Marks)**

b. Find the image of lines parallel to x – axis and lines parallel to y – axis under the transformation $w = z^2$. Draw neat sketch. **(07 Marks)**

c. Find the bilinear transformation that maps the points z = -1, j, 1 on to the points w = 1, j, -1. **(07 Marks)**

3 a. If f(z) is analytic within and on a simple closed curve C and 'a' is a point within 'C' then prove that $f(a) = \frac{1}{2\pi j}\int_c \frac{f(z)}{z-a}dz$. **(06 Marks)**

b. State Cauchy's residue theorem. Hence or otherwise evaluate – $\int_c \frac{e^{2z}}{(z+2)(z+4)(z+7)}dz$ for 'C' as $|Z| = 3$. **(07 Marks)**

c. Find the Taylor's series expansion of $f(z) = \frac{1}{(z+1)^2}$ about the point z = -i. **(07 Marks)**

4 a. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(06 Marks)**

b. Express polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(07 Marks)**

c. Compute P_0, P_1, P_2, P_3, P_4 using Rodrigue's formula. **(07 Marks)**

PART – B

5 a. Fit a parabola $y = a + bx + cx^2$, given the data : **(06 Marks)**

x	–3	–2	–1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

b. Obtain the coefficient of correlation and the liens of regression if : **(07 Marks)**

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

c. A tea set has four sets of cups and saucers. Two of these sets are of one colour and the other two sets are of different colours. (totally three colours). If the cups are placed randomly on saucers, what is the probability that no cup is on a saucer of same colour. **(07 Marks)**

6 a. Define i) Random variable ii) Discrete probability distribution with an example. **(06 Marks)**

b. The probability that a man aged 60 will live up to 70 is 0.65. What is the probability that out of 10 men, now aged 60, i) exactly 9, ii) at the most 9 iii) at least 7, will live up to the age of 70 years. **(07 Marks)**

c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation, given that A(0.5) = 0.19 and A(1.4) = 0.42. **(07 Marks)**

7 a. Find the probability that in 100 tosses of a fair coin between 45% and 55% of the outcomes are heads. **(06 Marks)**

b. A mechanist is making engine parts with axle diameter of 0.7 inches. A random sample of 10 parts showed a mean of 0.472 inches with a standard deviation of 0.04 inches. On the basis of this sample, can it be concluded that the work is inferior at 5% level of significance. **(07 Marks)**

c. For the following data test the hypothesis that the accidents are uniformly distributed over all the days of the week for 99% confidence.

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	16	8	12	11	9	14	84

(07 Marks)

8 a. Find the –
Marginal distribution of x
Marginal distribution of y
Cov (x, y) if the joint pdf of x and y is

x \ y	1	3	9
2	$1/8$	$1/24$	$1/12$
4	$1/4$	$1/4$	0
6	$1/8$	$1/24$	$1/12$

(06 Marks)

b. Find the fixed probability vector of regular stochastic matrix

$$A = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}.$$

(07 Marks)

c. A company executive changes his car every year. If he has a car of make A, he changes over to make B. from make B he changes over to make C. if he has car 'C' then he gives equal preference to change over to make A or make B car. If he had a car of make C in year 2008 find the probability that he will have a car of i) make A in 2010, ii) make 'C' in 2010. **(07 Marks)**

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06MAT41

Fourth Semester B.E. Degree Examination, Dec.09/Jan.10
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Employ Taylor's series method to find an approximate solution correct to fourth decimal places for the following initial value problem at $x = 0.1$, $dy/dx = x - y^2$, $y(0)=1$. **(06 Marks)**

b. Using modified Euler's method to find $y(0.1)$ given $dy/dx = x^2 + y$, $y(0) = 1$ by taking $h=0.05$. Perform two iterations in each step. **(07 Marks)**

c. If $dy/dx = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.04$ and $y(0.3)=2.09$ find $y(0.4)$ correct to four decimal places. By using Milne's predictor-corrector method (Use corrector formula twice). **(07 Marks)**

2 a. Derive Cauchy-Riemann equations in Cartesian form. **(06 Marks)**

b. Find the analytic function $f(z) = u+iv$ whose real part is $e^{-x}(x\cos y + y\sin y)$. **(07 Marks)**

c. Find the bilinear transformation which maps the points $Z=0, i, \infty$ onto the points $w = 1, -i, -1$ respectively. Find the invariant points. **(07 Marks)**

3 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ in terms of Laurent's series valid in the regions i) $|z-1| < 1$ ii) $|z-1| > 1$. **(07 Marks)**

c. Evaluate $\int_c \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$ using Cauchy's Residues theorem where c is the circle $|z| = 3$. **(07 Marks)**

4 a. Solve in series the equation $x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + xy = 0$ **(06 Marks)**

b. Solve Bessel's differential equation leading to $J_n(x)$. **(07 Marks)**

c. Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. **(07 Marks)**

PART – B

5 a. The pressure and volume of a gas are related by the equation $PV^v = K$, where v and K being constants. Fit this equation to the following set of observations. **(06 Marks)**

P (kg/cm²)	0.5	1.0	1.5	2.0	2.5	3.0
V (litre)	1.62	1.00	0.75	0.62	0.52	0.46

b. Find the correlation coefficient and the regression lines of y on x and x on y for the following data: **(07 Marks)**

x	1	2	3	4	5
y	2	5	3	8	7

c. State and prove Baye's theorem. **(07 Marks)**

6 a. The probability density function of a variate X is

X:	0	1	2	3	4	5	6
P(X):	k	3k	5k	7k	9k	11k	13k

Find i) k ii) $P(X \geq 5)$ iii) $P(3 < X \leq 6)$ **(06 Marks)**

b. The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are choosen at random, what is the probability that i) no line is busy ii) at least 5 lines are busy iii) at most 3 lines are busy. **(07 Marks)**

c. Obtain the mean and standard deviation of the normal distribution. **(07 Marks)**

7 a. Explain the following terms:
i) Null hypothesis
ii) Confidence limits
iii) Type I & Type II errors. **(06 Marks)**

b. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate that the die is biased? **(07 Marks)**

c. The nine items of a sample have the following values: 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? (Given $t_{0.05}$ for 8 df = 2.31). **(07 Marks)**

8 a. The joint probability distribution of two random variables X and Y are given below.

X \ Y	-3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Determine i) E(X) and E(Y) ii) E(XY) iii) COU (X,Y) **(06 Marks)**

b. Every year, a man trades his car for a new car. If he has a Maruti, he trades it for an Ambassador. If he has an Ambassador, he trades it for a Santro. However, if he has a Santro, he is just as likely to trade if for a new Santro as to trade if for Maruti or an Ambassador. In 2000, he bought his first car, which was Santro. Find the probability that he has
i) 2002 Santro ii) 2002 Maruti. **(07 Marks)**

c. Define stochastic matrix. Find the unique fixed probability vector for the regular stochastic matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$ **(07 Marks)**

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